Safety Quantification for Nonlinear and Time-Delay Systems using Occupation Measures

Author: **Jared Miller**

Committee: Octavia Camps
Didier Henrion (LAAS-CNRS)
Bahram Shafai
Eduardo Sontag
Mario Sznaier

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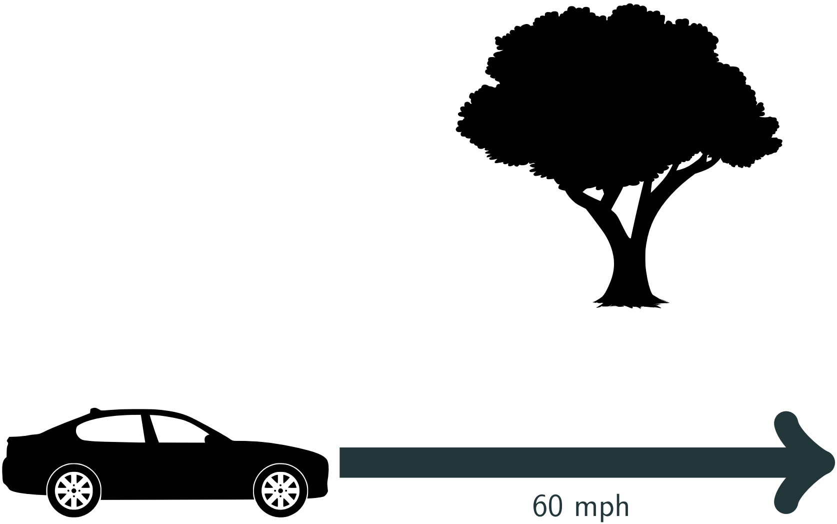
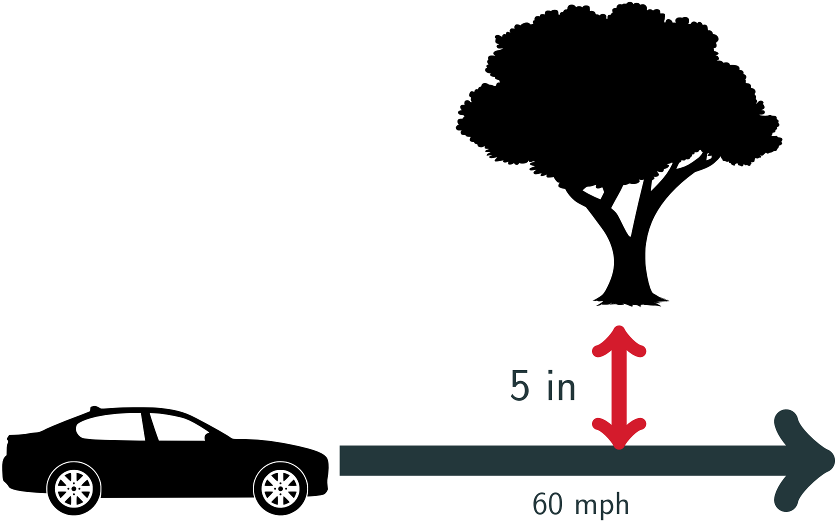
Northeastern University

Safety Example

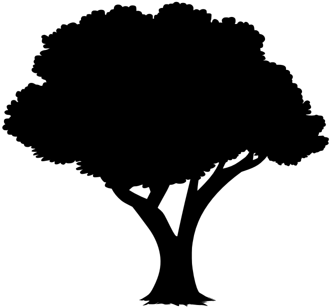


Safety Example (Barrier/Density Function)

Safety Example (Distance Estimate)



Safety Example



Safety Quantification

Motivation: Epidemic

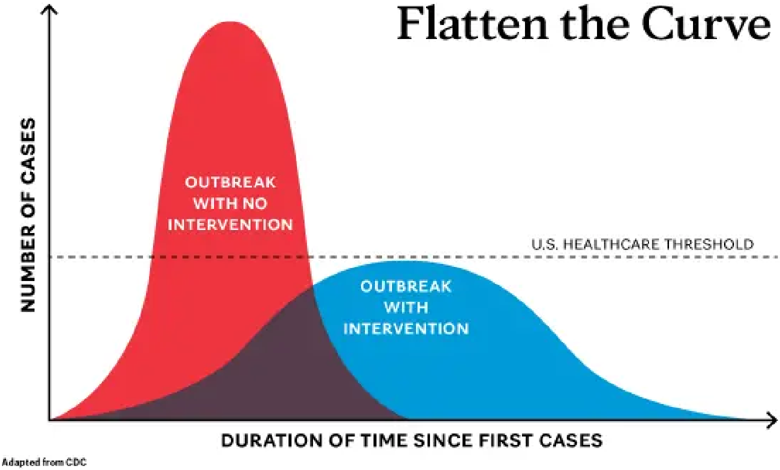
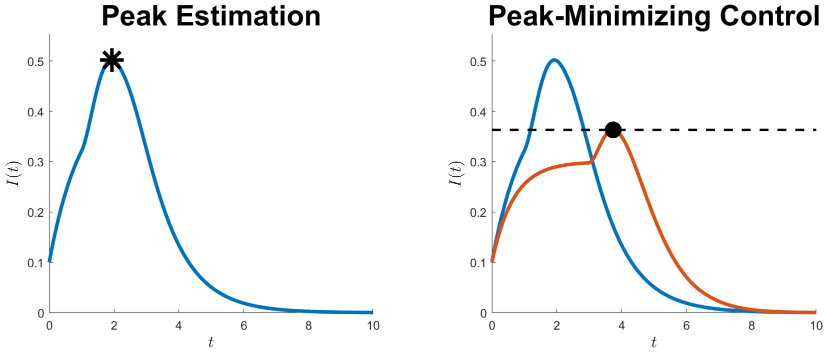


Image credit to Mayo Clinic News Network

Problems Covered



Main Ideas

Pose safety quantification problems

Want convex, convergent, bisection-free algorithms

Formulate using convex linear programs in measures

Increasing-quality bounds using Semidefinite Programming

Overview of Presentation

Peak estimation background

1. Survey of Thesis Work
2. Peak Value-at-Risk Estimation
3. Time-Delay Systems

Wrap-up

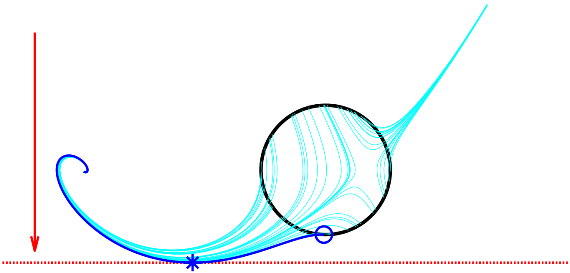
Peak Estimation Background

Peak Estimation Background

Find extreme value of $p(x)$ along trajectories

$$P^* = \sup_{t,\ x_0 \in X_0} p(x(t \mid x_0))$$

$$\dot{x}(t) = f(t, x(t)) \qquad \forall t \in [0, T], \qquad x(0) = x_0.$$



$$p(x) = -x_2, \qquad \dot{x} = [x_2, -x_1 - x_2 + x_1^3/3]$$

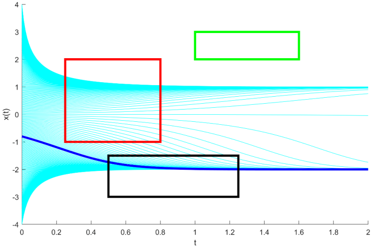
Occupation Measure

Time trajectories spend in set

Test function
$v(t, x) \in C([0, T] \times X)$

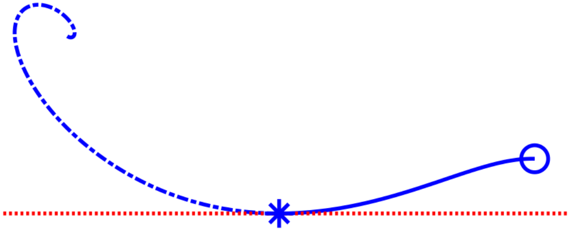
Single trajectory:
$\langle v, \mu \rangle = \int_0^T v(t, x(t \mid x_0))dt$

Averaged trajectory: $\langle v, \mu \rangle = \int_X \left(\int_0^T v(t, x)dt \right) d\mu_0(x)$



$x' = -x(x + 2)(x - 1)$

Connection to Measures



Measures: Initial μ_0, Peak μ_p, Occupation μ

For all functions $v(t,x) \in C([0,T] \times X)$

$$\begin{aligned}
\mu_0^* : &\qquad \langle v(0,x), \mu_0^* \rangle = v(0, x_0^*) \\
\mu_p^* : &\qquad \langle v(t,x), \mu_p^* \rangle = v(t_p^*, x_p^*) \\
\mu^* : &\qquad \langle v(t,x), \mu^* \rangle = \int_0^{t_p^*} v(t, x^*(t \mid x_0^*)) dt
\end{aligned}$$

Liouville Equation

Lie derivative (instantaneous change along f) $\forall v \in C^1$:

$$\mathcal{L}_f v = \partial_t v(t,x) + f(t,x) \cdot \nabla_x v(t,x) \tag{1a}$$

Conservation law: final = initial + accumulated change

$$\langle v(t,x), \mu_p \rangle = \langle v(0,x), \mu_0 \rangle + \langle \mathcal{L}_f v(t,x), \mu \rangle \tag{1b}$$

$$\mu_p = \delta_0 \otimes \mu_0 + \mathcal{L}_f^\dagger \mu \tag{1c}$$

Liouville 'represents' dynamics $\dot{x}(t) = f(t, x(t))$

Measures for Peak Estimation

Infinite-dimensional Linear Program (Cho, Stockbridge, 2002)

$$p^* = \sup \quad \langle p(x), \mu_p \rangle \tag{2a}$$
$$\langle 1, \mu_0 \rangle = 1 \tag{2b}$$
$$\langle v(t,x), \mu_p \rangle = \langle v(0,x), \mu_0 \rangle + \langle \mathcal{L}_f v(t,x), \mu \rangle \quad \forall v \tag{2c}$$
$$\mu, \mu_p \in \mathcal{M}_+([0,T] \times X) \tag{2d}$$
$$\mu_0 \in \mathcal{M}_+(X_0) \tag{2e}$$

Instance of Optimal Control Program (Lewis and Vinter, 1980)

$(\mu_0^*, \mu_p^*, \mu^*)$ is feasible with $P^* = \langle p(x), \mu_p^* \rangle \leq p^*$

$P^* = p^*$ if compactness, Lipschitz properties hold

Moments for Peak Estimation

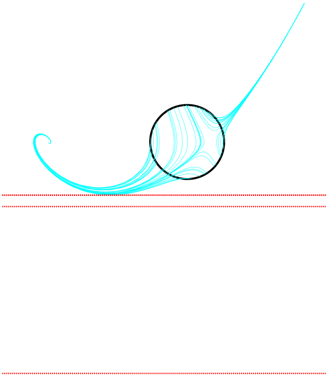
Moment: $y_\alpha = \langle x^\alpha, \nu \rangle \; \forall \alpha \in \mathbb{N}^n$

Moment matrix $\mathbb{M}[y]_{\alpha\beta} = y_{\alpha+\beta}$ is PSD

$$\mathbb{M}_2[y] = \begin{bmatrix} y_{00} & y_{10} & y_{01} & y_{20} & y_{11} & y_{02} \\ y_{10} & y_{20} & y_{11} & y_{30} & y_{21} & y_{12} \\ y_{01} & y_{11} & y_{02} & y_{21} & y_{12} & y_{03} \\ y_{20} & y_{30} & y_{21} & y_{40} & y_{31} & y_{11} \\ y_{11} & y_{21} & y_{12} & y_{31} & y_{22} & y_{13} \\ y_{02} & y_{12} & y_{03} & y_{22} & y_{13} & y_{04} \end{bmatrix} \succeq 0$$

Liouville induces affine relation in $(\mu^0, \mu^p, \mu) \to (y^0, y^p, y)$

Peak Estimation Example Bounds



Converging bounds to min. $x_2 = -0.5734$ (moment-SOS)

Box region $X = [-2.5, 2.5]$, time $t \in [0, 5]$

Max. PSD size: $\binom{(n+1)+(d+\lfloor \deg f/2 \rfloor)}{n+1}$ (Fantuzzi, Goluskin, 2020)

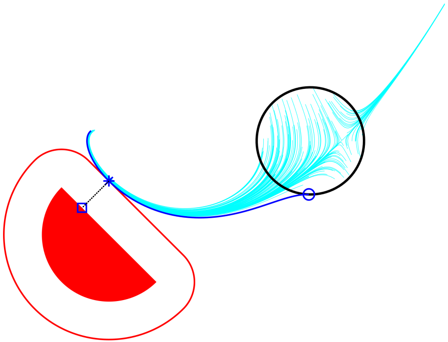
Survey of Thesis Work

Distance Estimation Problem

Unsafe set X_u, point-set distance $c(x; X_u) = \inf_{y \in X_u} c(x, y)$

$$P^* = \inf_{t,\, x_0 \in X_0} c(x(t \mid x_0); X_u)$$

$$\dot{x}(t) = f(t, x(t)) \qquad \forall t \in [0, T], \quad x(0) = x_0.$$



L_2 bound of 0.2831

Distance Program (Measures)

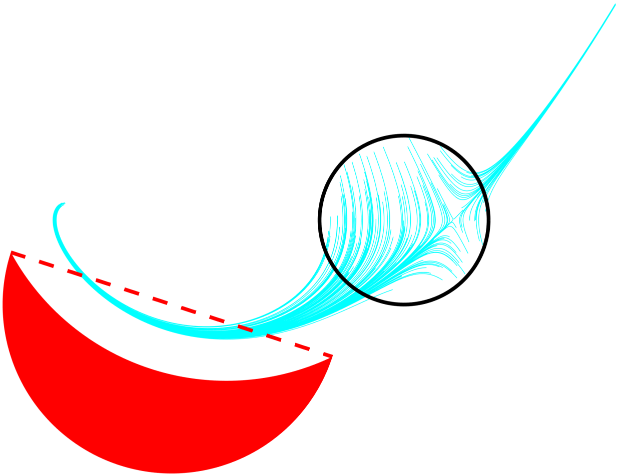
Infinite Dimensional Linear Program (Convergent)

$$
\begin{aligned}
p^* = \ & \inf \quad \langle c(x,y), \eta(x,y) \rangle && && \text{(3a)}\\
& \langle 1, \mu_0 \rangle = 1 && && \text{(3b)}\\
& \langle v(t,x), \mu_p \rangle = \langle v(0,x), \mu_0 \rangle + \langle \mathcal{L}_f v(t,x), \mu \rangle && \forall v && \text{(3c)}\\
& \langle w(x), \eta(x,y) \rangle = \langle w(x), \mu_p(t,x) \rangle && \forall w && \text{(3d)}\\
& \eta \in \mathcal{M}_+(X \times X_u) && && \text{(3e)}\\
& \mu_p, \ \mu \in \mathcal{M}_+([0,T] \times X) && && \text{(3f)}\\
& \mu_0 \in \mathcal{M}_+(X_0) && && \text{(3g)}
\end{aligned}
$$

Probability measures: (μ_0, μ_p, η)

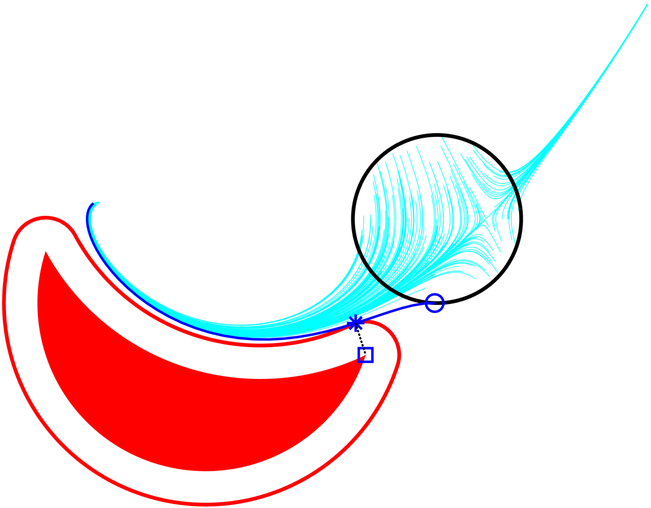
Near-optimal trajectories if moment-matrix $\approx$ rank-1

Distance Example (Flow Moon)



Collision if X_u was a half-circle

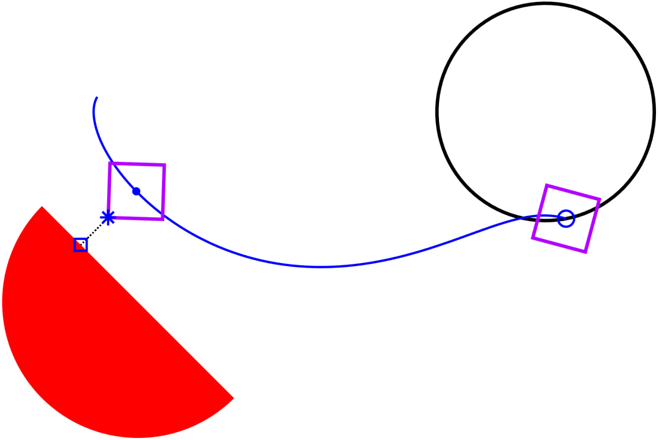
Distance Example (Flow Moon)



L_2 bound of 0.1592

Safety of Shapes

Points on shape S with orientation ω (e.g., rigid body motion)

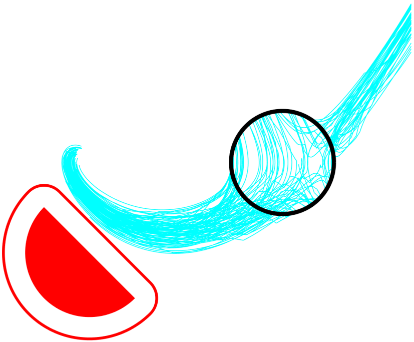


L_2 bound of 0.1465, rotating square

Distance with Bounded Uncertainty

Dynamics $\dot{x}(t) = f(t, x(t), w(t))$ with $w(t) \in W$

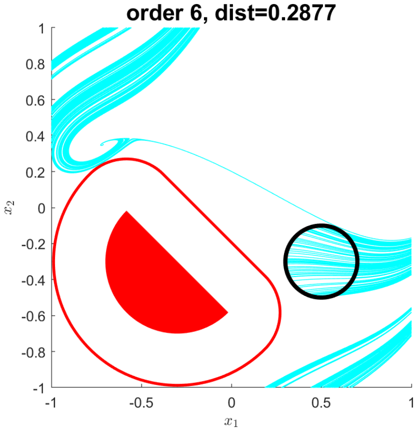
Young measure $\mu(t, x, w)$, Liouville term $\langle \mathcal{L}_f v(t, x, w), \mu \rangle$



L_2 bound of 0.1691, $w(t) \in [-1, 1]$

Hybrid Systems

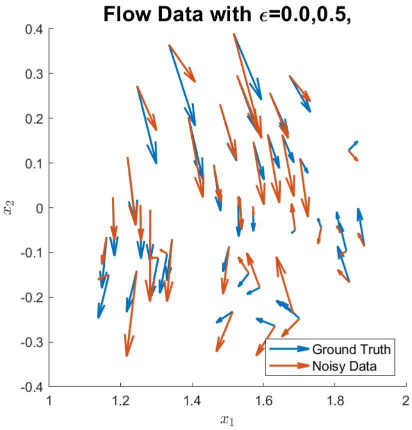
Continuous dynamics with discrete jumps/transitions



$$R_{\text{left}\to\text{bottom}} = [1 - x_2; x_1], \qquad R_{\text{right}\to\text{top}} = [x_2; x_1]$$

Sampling: Flow System

Data $\mathcal{D} = \{(t_j, x_j, \dot{x}_j)\}_j$ under mixed L_∞-bounded noise



$$\dot{x} = [x_2, -x_1 - x_2 + x_1^3/3]$$

Dynamics Model

Given data $\mathcal{D}$, budget ϵ, system model $\{f_0, f_\ell\}$

Parameterize ground truth F by functions in dictionary

$$\dot{x}(t) = f(t, x, w) = f_0(t, x) + \sum_{\ell=1}^{L} w_\ell f_\ell(t, x)$$

Ground truth satisfies corruption $J(w^*) \leq \epsilon$

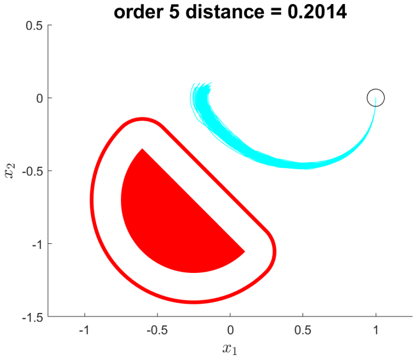
L_∞ example: $J(w) = \max_j \|f(t_j, x_j, w) - \dot{x}_j\|_\infty$

Distance Estimation Example (Flow)

Input-affine + Semidefinite Representable uncertainty

$$\mathcal{L}_f v(t, x, w) \leq 0 \qquad \forall (t, x, w) \in [0, T] \times X \times W$$

PSD Size 8568 → 56 ($L = 10$) using robust counterparts



$$\dot{x} = [x_2,\ \text{cubic}(x_1, x_2)]$$

Crash-Safety

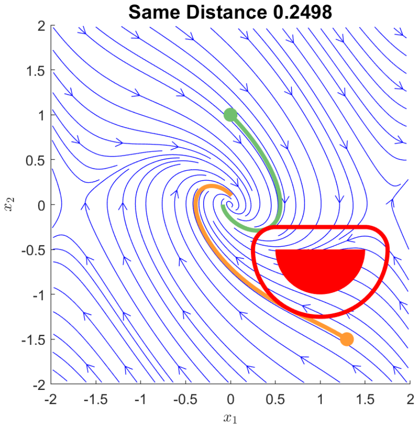
How much data corruption is needed to crash?

$$
\begin{aligned}
Q^* = \inf_{t^*,\ x_0,\ w} & \left[\sup_{t \in [0, t^*]} J(w(t)) \right] \\
& \dot{x}(t) = f(t, x(t), w(t)) \qquad \qquad \forall t \in [0, t^*] \\
& x(t \mid x_0, w(\cdot)) \in X_u \\
& w(\cdot) \in W,\ t^* \in [0, T],\ x_0 \in X_0
\end{aligned}
$$

Model safe if $Q^* > \epsilon$

Example Crash-Bounds

Two trajectories have same distance, different crash-bounds



Green-Top $Q^* = 0.316$, Yellow-Bottom $Q^* = 0.622$

Peak-Minimizing Control

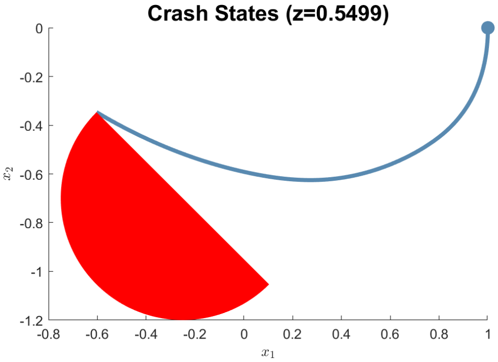
Add state $\dot{z} = 0$ (Molina, Rapaport, Ramírez 2022)

$$
\begin{aligned}
Q_z^* = \inf_{t^*,\ x_0,\ z,\ w} \quad & z && && \text{(4a)} \\
& \dot{x}(t) = f(t, x(t), w(t)) && \forall t \in [0, t^*] && \text{(4b)} \\
& \dot{z}(t) = 0 && \forall t \in [0, t^*] && \text{(4c)} \\
& J(w(t)) \leq z && \forall t \in [0, t^*] && \text{(4d)} \\
& x(t^* \mid x_0, w(\cdot)) \in X_u && && \text{(4e)} \\
& w(\cdot) \in W, \ \ t^* \in [0, T] && && \text{(4f)} \\
& x_0 \in X_0, z \in [0, J_{\max}] && && \text{(4g)}
\end{aligned}
$$

Equivalent formulation, $Q^* = Q_z^*$

Data-Driven Flow Crash-Bound

CasADi matches degree-4 moment-SOS crash bound



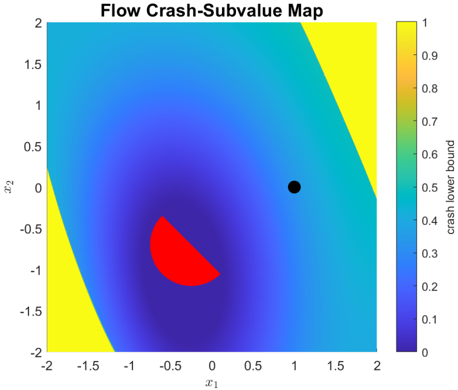
Terminal measure $\mu_p \in \mathcal{M}_+([0, T] \times X_u)$

True $\epsilon = 0.5 < 0.5499$, distance ≈ 0.2014

Flow Crash-Subvalue

Piecewise-polynomial subvalue for crash-safety

Based on Joint+Marginal optimization (Lasserre, 2010)



Bound of $0.3399 \leq 0.5499$, but valid everywhere in X

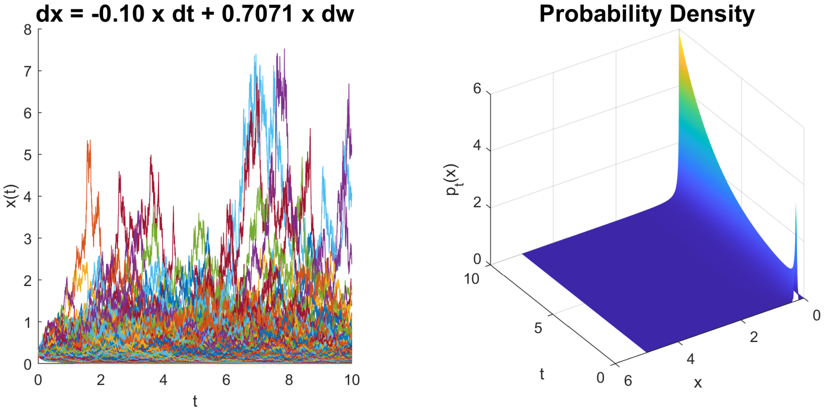
Peak Value-at-Risk Estimation

with M. Tacchi, M. Sznaier, A. Jasour

Stochastic Differential Equation

Multivariate SDE $dx = f(t,x)dt + g(t,x)dw$ (Itô)

Drift f and Diffusion g

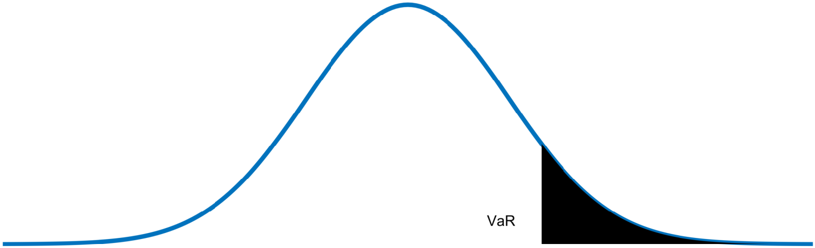


Geometric Brownian Motion

Value-at-Risk (Quantile)

ϵ-VaR of univariate measure $\omega(q)$ is unique number with

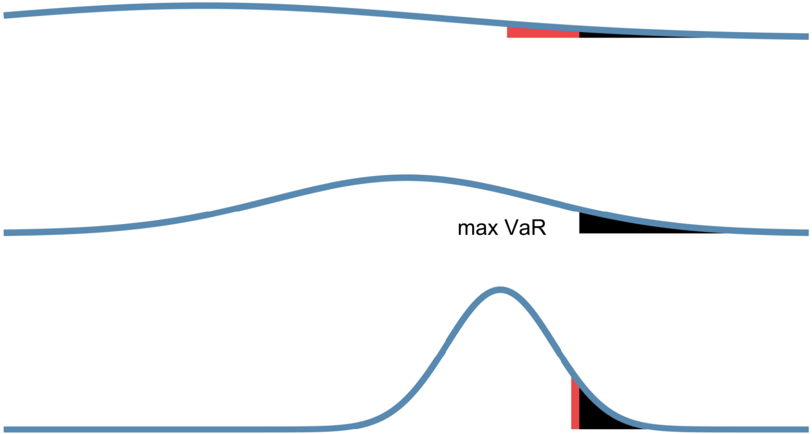
$$\text{Prob}_\omega(q \geq VaR_\epsilon(\omega)) = \epsilon$$



$VaR = 1.282$ for unit normal distribution at $\epsilon = 10\%$

Maximal Value at Risk

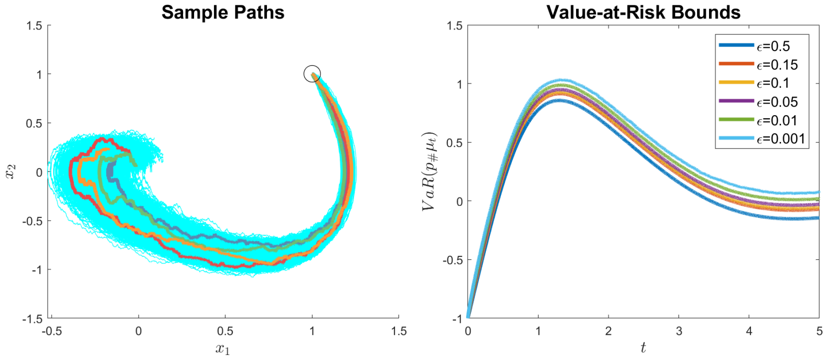
Maximize ϵ-VaR among multiple distributions



Red + Black areas = 10% probability

Value-at-Risk Example (Monte Carlo)

50,000 samples with $T = 5,\ \Delta t = 10^{-3}$



VaR of $p = -x_2$ along $dx = \begin{bmatrix} x_2 \\ -x_1 - x_2 - \frac{1}{2}x_1^3 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} dw$

Chance-Peak Problem

Maximize VaR of $p(x)$ along SDE trajectories

$p_\# \mu_{t^*}$: distribution of $p(x(t))$ at time t^*

$$
\begin{aligned}
P^* = \sup_{t^* \in [0, T]} \ & VaR_\epsilon(p_\# \mu_{t^*}) && \text{(5a)} \\
& dx = f(t, x)dt + g(t, x)dw && \text{(5b)} \\
& \quad \text{stopping time of } \min(t^*, \text{exit from } X) && \text{(5c)} \\
& x(0) \sim \mu_0. && \text{(5d)}
\end{aligned}
$$

Value-at-Risk Bounds

Concentration inequalities can upper-bound VaR

$$\mathit{VaR}_\epsilon(\omega) \leq \mathrm{stdev}(\omega) r + \mathrm{mean}(\omega)$$

Name	r	Condition
Cantelli	$\sqrt{1/(\epsilon) - 1}$	ω probability distribution
VP	$\sqrt{4/(9\epsilon) - 1}$	ω unimodal, $\epsilon < 1/6$

Coherent Risk Measures (e.g., CVaR) can also bound VaR

Concentration-Bounded Chance-Peak

Apply concentration inequalities to get upper bound $P_r^* \geq P^*$

Objective upper-bounds VaR w.r.t. time-t^* distribution μ_{t^*}

$$P_r^* = \sup_{t^* \in [0,T]} r\sqrt{\langle p^2, \mu_{t^*}\rangle - \langle p, \mu_{t^*}\rangle^2} + \langle p, \mu_{t^*}\rangle \tag{6a}$$

$$dx = f(t,x)dt + g(t,x)dw \tag{6b}$$

$$\text{stopping time of } \min(t^*, \text{exit from } X) \tag{6c}$$

$$x(0) \sim \mu_0. \tag{6d}$$

Max-Mean: $\epsilon = 0.5$, $r = 0$ (Cho, Stockbridge, 2002)

Occupation Measure Formulation

Occupation measure μ, terminal measure μ_τ

Second-Order Cone Program in measures (3d SOC)

$$p_r^* = \sup r\sqrt{\langle p^2, \mu_\tau \rangle - \langle p, \mu_\tau \rangle^2} + \langle p, \mu_\tau \rangle \tag{7a}$$

$$\mu_\tau = \delta_0 \otimes \mu_0 + \mathcal{L}^\dagger \mu \tag{7b}$$

$$\mu_\tau, \; \mu \in \mathcal{M}_+([0, T] \times X) \tag{7c}$$

Generator $\mathcal{L}v = \partial_t v + f \cdot \nabla_x v + g^T(\nabla^2_{xx} v)g/2$ (Dynkin's)

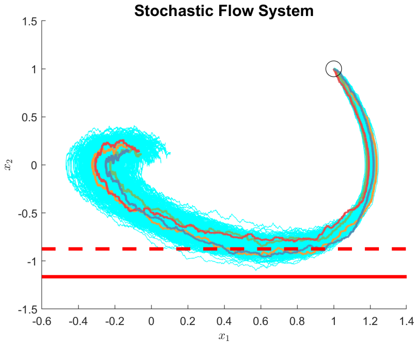
Results in upper-bound $p_r^* \geq P_r^* \geq P^*$, use moments

Chance-Peak Examples

Two-State

Stochastic Flow system from Prajna, Rantzer with $T = 5$

$$dx = \begin{bmatrix} x_2 \\ -x_1 - x_2 - \frac{1}{2}x_1^3 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} dw.$$

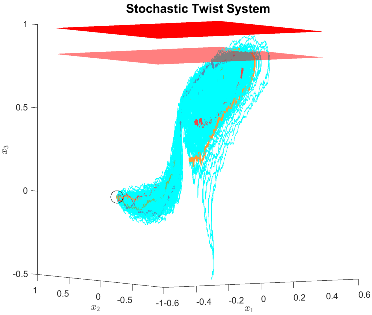


Maximize $-x_2$ with $d = 6$ (dashed=50%, solid=85% [ours])

Three-State

Stochasic Twist system with $T = 5$

$$dx = \begin{bmatrix} -2.5x_1 + x_2 - 0.5x_3 + 2x_1^3 + 2x_3^3 \\ -x_1 + 1.5x_2 + 0.5x_3 - 2x_2^3 - 2x_3^3 \\ 1.5x_1 + 2.5x_2 - 2x_3 - 2x_1^3 - 2x_2^3 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} dw.$$

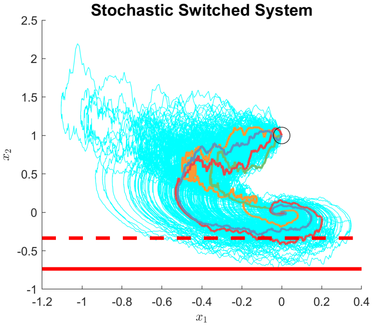


Maximize x_3 with $d = 6$ (translucent=50%, solid=85%)

Two-State Switching

Switching subsystems at $T = 5$

$$dx = \left\{ \begin{bmatrix} -2.5x_1 - 2x_2 \\ -0.5x_1 - x_2 \end{bmatrix}, \begin{bmatrix} -x_1 - 2x_2 \\ 2.5x_1 - x_2 \end{bmatrix} \right\} dt + \begin{bmatrix} 0 \\ 0.25x_2 \end{bmatrix} dw$$

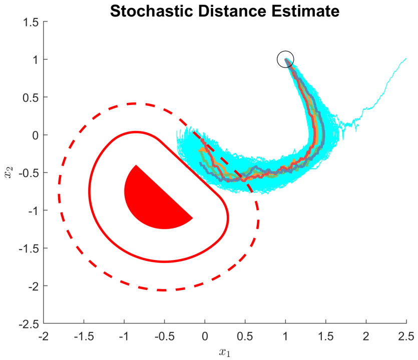


Maximize $-x_2$ with $d = 6$ (dashed=50%, solid=85%)

Two-State Distance

Half-circle unsafe set X_u

Based on distance estimation program



Minimize L_2 distance to X_u with $d = 6$ (dashed=50%, solid=85%)

Time-Delay Peak Estimation

with M. Korda, V. Magron, M. Sznaier

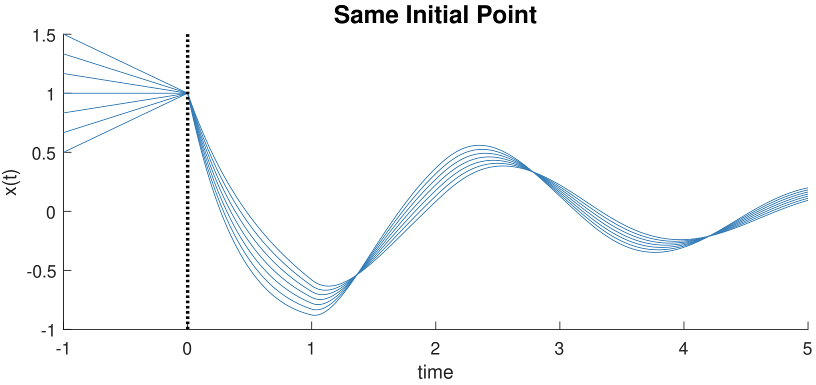
Time-Delay Examples

Delay between state change and its effect on system

$$\dot{x}(t) = f(t, x(t), x(t-\tau)) \qquad \forall t \in [0, T]$$
$$x(s) = x_h(s) \qquad \forall s \in [-\tau, 0]$$

System	**Delay**
Epidemic	Incubation Period
Population	Gestation Time
Traffic	Reaction Time
Congestion	Queue Time
Fluid Flow	Moving in Pipe

Dependence on History

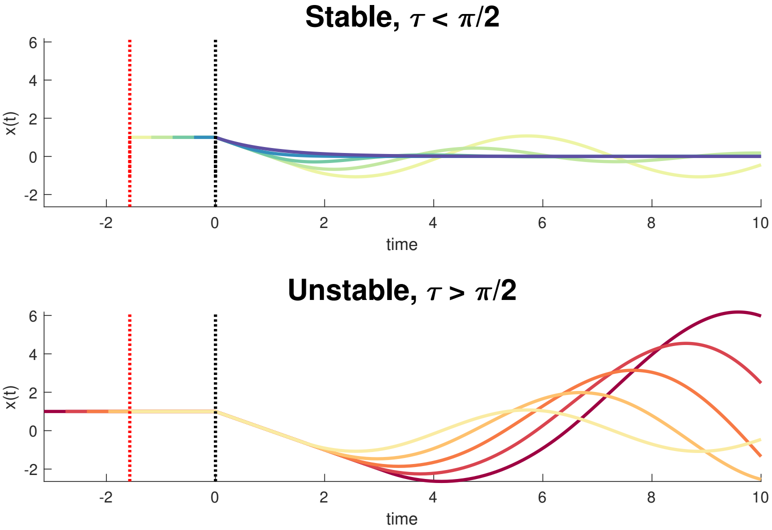


$$x'(t) = -2x(t) - 2x(t-1)$$

All trajectories pass through $(t, x) = (0, 1)$

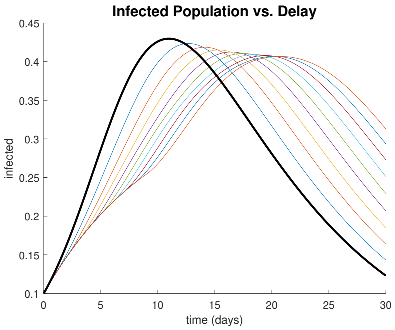
Initial history determines behavior, not just initial point

Delay Bifurcation Example

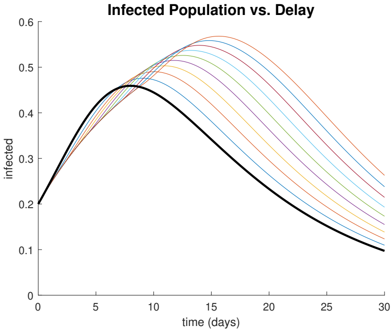


$x'(t) = -x(t-\tau)$ (Fridman 2014)

Peak Value vs. Delay



(a) $I_h = 0.1$, peak decreases



(b) $I_h = 0.2$, peak increases

$$\begin{bmatrix} S'(t) \\ I'(t) \end{bmatrix} = \begin{bmatrix} -0.4S(t)I(t) \\ 0.4S(t-\tau)I(t-\tau) - 0.1I(t) \end{bmatrix}$$

Peak Estimation of Time-Delay Systems

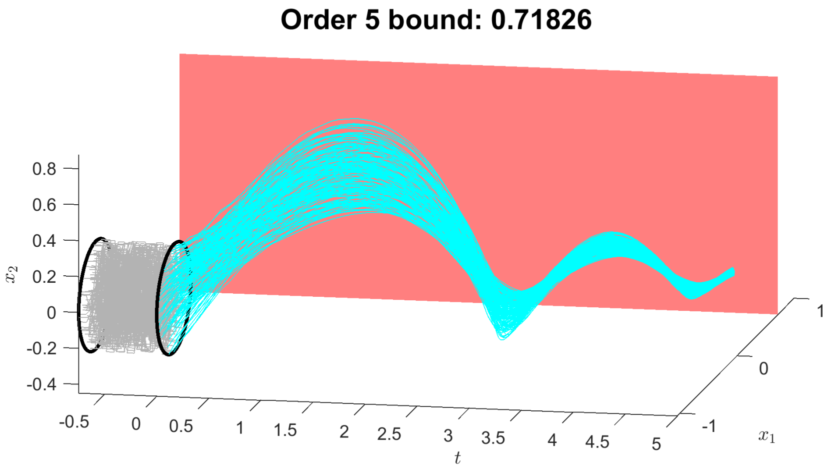
History $x_h(t)$ resides in a class of functions $\mathcal{H}$

Graph-constrained $\mathcal{H}$: $(t, x_h(t))$ contained in $H_0 \subset [-\tau, 0] \times X$

$$
\begin{aligned}
P^* = \sup_{t^*,\, x_h} \ & p(x(t^*)) \\
& \dot{x} = f(t, x(t), x(t-\tau)) && t \in [0, t^*] \\
& x(t) = x_h(t) && t \in [-\tau, 0] \\
& x_h(\cdot) \in \mathcal{H}
\end{aligned}
$$

Represent $x(t \mid x_h) : t \in [-\tau, t^*]$ as occupation measure

Time-Varying Preview



Maximize x_1 on $\dot{x}(t) = \begin{bmatrix} x_2(t)t - 0.1x_1(t) - x_1(t-\tau)x_2(t-\tau) \\ -x_1(t)t - x_2(t) + x_1(t)x_1(t-\tau) \end{bmatrix}$

Existing Methods (very brief)

Certificates of Stability

- Lyapunov-Krasovskii
- Razumikhin
- LMI, Wirtinger
- ODE-Transport PDE

Relaxed control (Warga 1974, Vinter and Rosenblueth 1991-2)

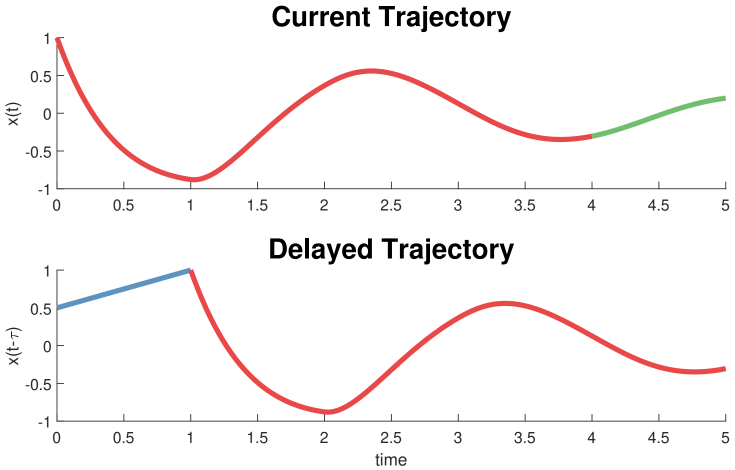
Fixed-terminal-time OCP with gridding (Barati 2012)

SOS Barrier (Papachristodoulou and Peet, 2010)

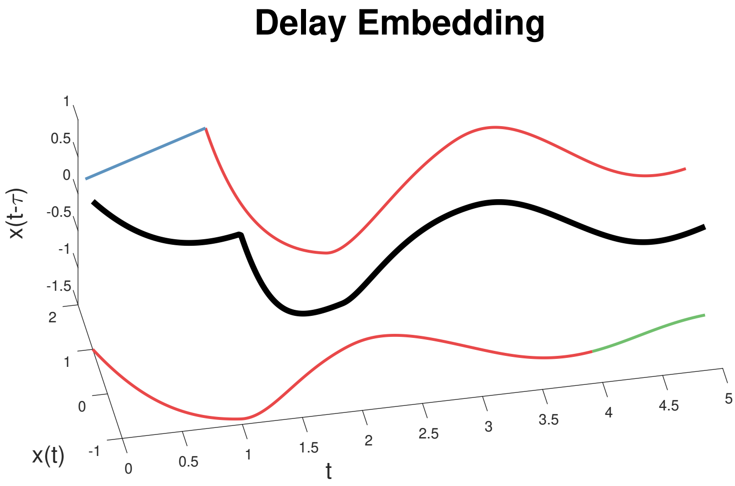
Riesz Operators (Magron and Prieur, 2020)

Time-Delay Measure Program

Time-Delay Visualization



$$x(t) = -2x(t) - 2x(t-1), \qquad x_h(t) = 1 - t/2$$



Black curve: $(t, x(t), x(t-\tau))$

Measure-Valued Solution

Tuple of measures for the delayed case

Peak	$\mu_p \in \mathcal{M}_+([0, T] \times X)$
Initial	$\mu_0 \in \mathcal{M}_+(X_0)$
History	$\mu_h \in \mathcal{M}_+(H_0)$
Occupation Start	$\bar{\mu}_0 \in \mathcal{M}_+([0, T - \tau] \times X^2)$
Occupation End	$\bar{\mu}_1 \in \mathcal{M}_+([T - \tau, T] \times X^2)$
Time-Slack	$\nu \in \mathcal{M}_+([0, T] \times X)$

Types of Constraints

Initial Conditions

Liouville: Dynamics

Consistency: Time-delay overlaps

Initial Conditions

Point evaluation $\langle 1, \mu_0 \rangle = 1$ at time $t = 0^+$

History $(t, x_h(t))$ defines a curve $[-\tau, 0]$, point at $x_h(0)$

t-marginal of μ_h should be the Lebesgue measure in $[-\tau, 0]$

Liouville

Treat $x(t-\tau) = x_1$ as an external input $\dot{x}_0 = f(t, x_0, x_1)$

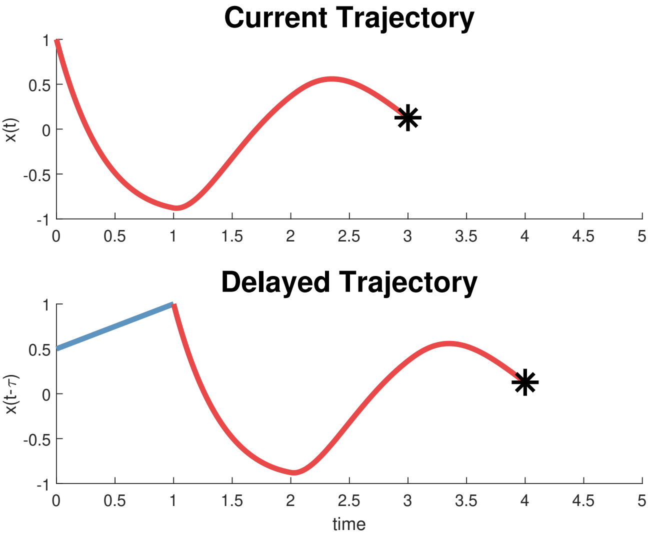
Sum $\bar{\mu} = \bar{\mu}_0 + \bar{\mu}_1$ in times $[0, T-\tau] \cap [T-\tau, T] = [0, T]$

Based on the delay embedding $(t, x(t), x(t-\tau))$

For all test functions $v \in C^1([0, T] \times X)$:

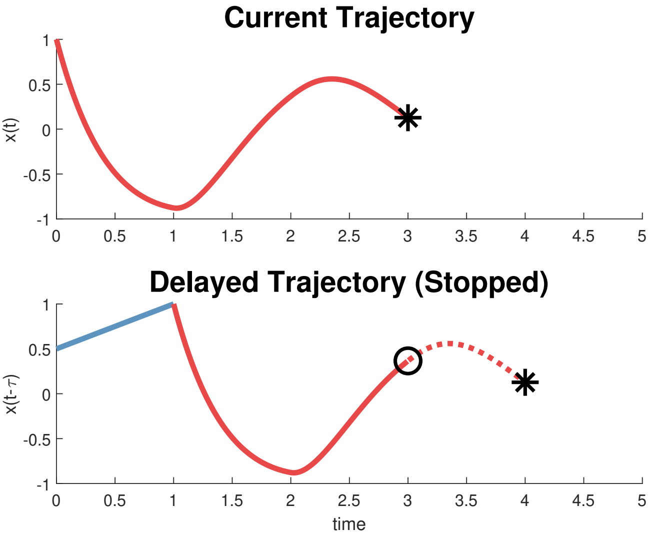
$$\langle v, \mu_p \rangle = \langle v(0, x), \mu_0(x) \rangle + \langle \mathcal{L}_{f(t,x_0,x_1)} v(t, x_0), \bar{\mu}(t, x_0, x_1) \rangle$$

Consistency Issue



Inconsistent elapsed times

Consistency Fix



Early stopping in delayed time

Consistency Constraint

Inspired by changing limits of integrals $t' \leftarrow t - \tau$

$$\left(\int_0^{t^*} + \int_{t^*}^{\min(T, t^*+\tau)}\right) \phi(t, x(t-\tau))dt$$
$$= \left(\int_{-\tau}^{0} + \int_{0}^{\min(t^*, T-\tau)}\right) \phi(t'+\tau, x(t'))dt'.$$

Shift-push $S^\tau_\#$ with $\langle \phi, S^\tau_\# \mu \rangle = \langle S^\tau \phi, \mu \rangle = \langle \phi(t+\tau, x), \mu \rangle$

Consistency constraint with time-slack ν

$$\pi^{tx_1}_\#(\bar{\mu}_0 + \bar{\mu}_1) + \nu = S^\tau_\#(\mu_h + \pi^{tx_0}_\# \bar{\mu}_0).$$

Measure Linear Program

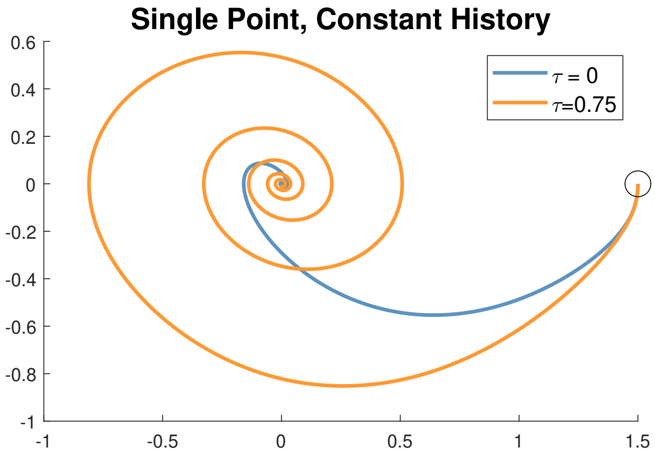
Linear program for time-delay peak estimation

$$
\begin{aligned}
p^* = \sup \quad & \langle p, \mu_p \rangle && \text{(8a)}\\
& \text{History-Validity}(\mu_0, \mu_h) && \text{(8b)}\\
& \text{Liouville}(\mu_0, \mu_p, \bar{\mu}_0, \bar{\mu}_1) && \text{(8c)}\\
& \text{Consistency}(\mu_h, \bar{\mu}_0, \bar{\mu}_1, \nu) && \text{(8d)}\\
& \text{Measure Definitions for } (\mu_h, \mu_0, \mu_p, \bar{\mu}_0, \bar{\mu}_1, \nu) && \text{(8e)}
\end{aligned}
$$

Largest measures $\bar{\mu}_0, \bar{\mu}_1$ have $2n + 1$ variables

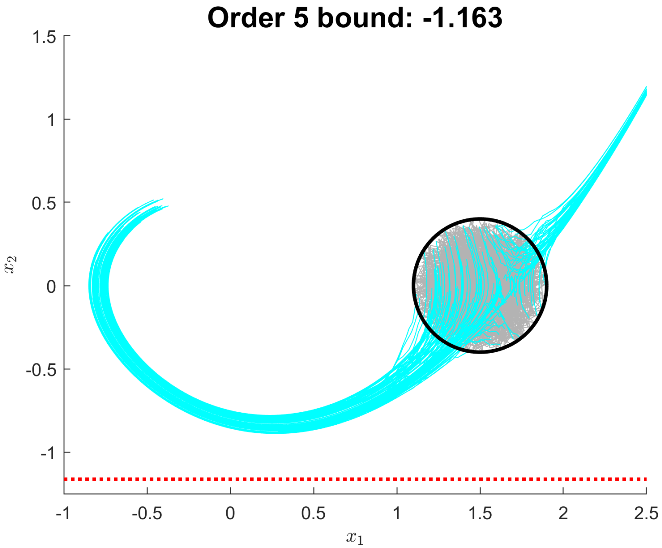
Time-Delay Examples

Delay Comparision



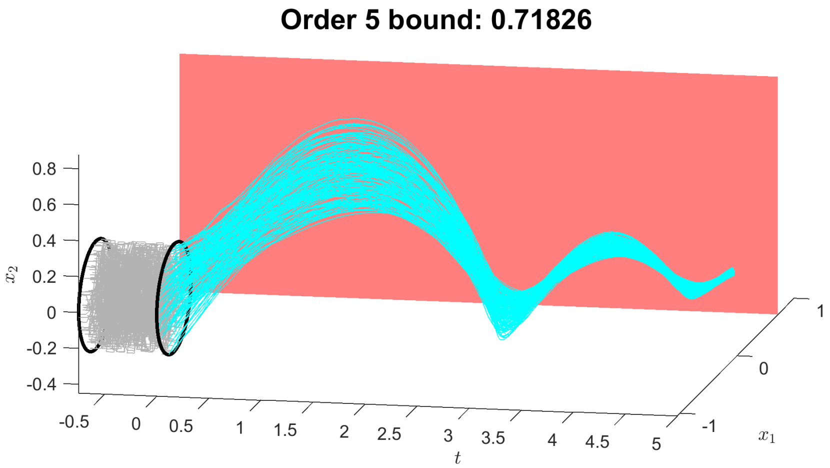
$$\dot{x}(t) = \begin{bmatrix} x_2 \\ -x_1(t-\tau) - x_2(t) + x_1(t)^3/3 \end{bmatrix}$$

Delayed Flow System



Minimize x_2 on the delayed Flow system

Time-Varying System (Reprise)



Maximize x_1 on $\dot{x}(t) = \begin{bmatrix} x_2(t)t - 0.1x_1(t) - x_1(t-\tau)x_2(t-\tau) \\ -x_1(t)t - x_2(t) + x_1(t)x_1(t-\tau) \end{bmatrix}$

Take-aways

Summary

Noted importance of safety quantification

Extended occupation measure methods for peak estimation

Performed data-driven analysis using robust counterparts

Adapted to non-ODE systems (Hybrid, SDE, Time-Delay)

Future Work

- No-relaxation-gap for chance-peak and time-delay system
- High-order concentration inequalities
- Other time-delay models
- Lévy processes, Poisson jumps
- Distance-maximizing control
- Increased scalability, robotic systems
- Real-time computation

Safety is Important

Quantify using Peak Estimation

Journal Papers

Published:

1. J. Miller, D. Henrion, and M. Sznaier, “Peak Estimation Recovery and Safety Analysis,” *IEEE Control Systems Letters*, vol. 5, no. 6, pp. 1982–1987, 2021 [link]

Conditionally Accepted:

1. J. Miller and M. Sznaier, “Bounding the Distance to Unsafe Sets with Convex Optimization,” (Conditionally accepted by IEEE Transactions on Automatic Control in 2022) [link]

Conference Proceedings

1. J. Miller and M. Sznaier, "Bounding the Distance of Closest Approach to Unsafe Sets with Occupation Measures," in *2022 61st IEEE Conference on Decision and Control (CDC)*, pp. 5008–5013, 2022. [link]
2. J. Miller and M. Sznaier, "Facial Input Decompositions for Robust Peak Estimation under Polyhedral Uncertainty," *IFACPapersOnLine*, vol. 55, no. 25, pp. 55–60, 2022. [link]. IFAC Young Author Award (ROCOND)
3. J. Miller, D. Henrion, M. Sznaier, and M. Korda, "Peak Estimation for Uncertain and Switched Systems," in *2021 60th IEEE Conference on Decision and Control (CDC)*, pp. 3222–3228, 2021. [link]. Outstanding Student Paper Award (CDC 2021)

Preprints

1. J. Miller, M. Korda, V. Magron, and M. Sznaier "Peak Estimation of Time Delay Systems using Occupation Measures, " 2023. [link]
2. J. Miller, M. Tacchi, M. Sznaier, and A. Jasour, "Peak Value-at-Risk Estimation for Stochastic Differential Equations using Occupation Measures," 2023. [link]
3. J. Miller and M. Sznaier, "Peak Estimation of Hybrid Systems with Convex Optimization, " 2023. [link]
4. J. Miller and M. Sznaier "Quantifying the Safety of Trajectories using Peak-Minimizing Control, " 2023. [link]
5. J. Miller and M. Sznaier, "Analysis and Control of Input-Affine Dynamical Systems using Infinite-Dimensional Robust Counterparts," 2023. [link]

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Last but not least

The Warden

Thank you again for your attention

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Cookies in Dana 429 (RSL)