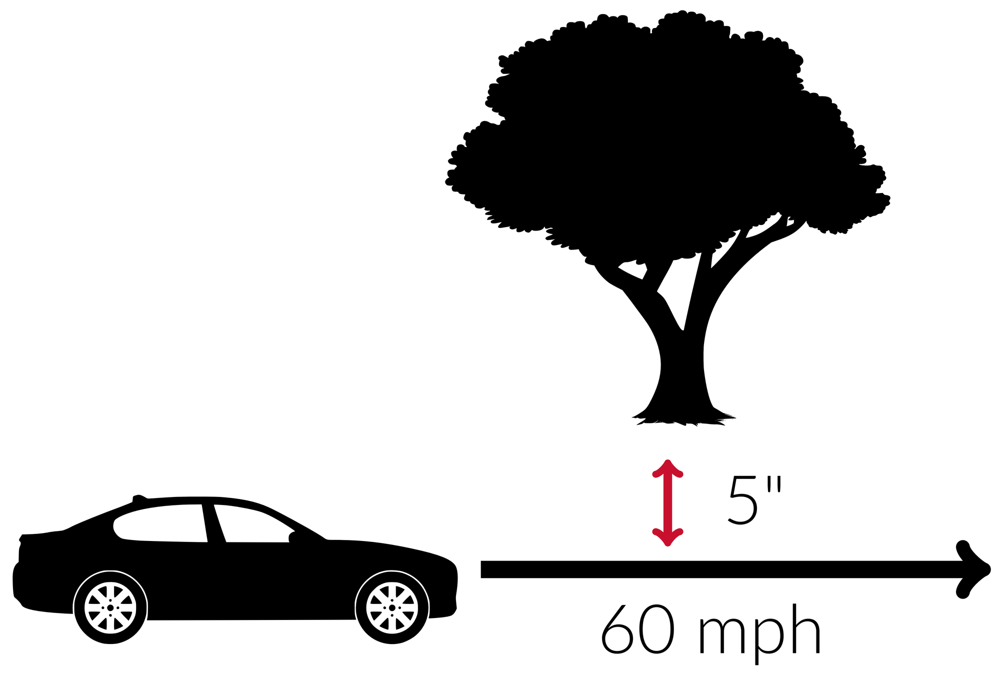
# Safety Analysis and Control using Measures

**Jared Miller**, Mario Sznaier

Robust Systems Lab, Electrical and Computer Engineering

## Motivation

Quantify the safety of trajectories




Use convex optimization to compute converging distance-lower-bounds

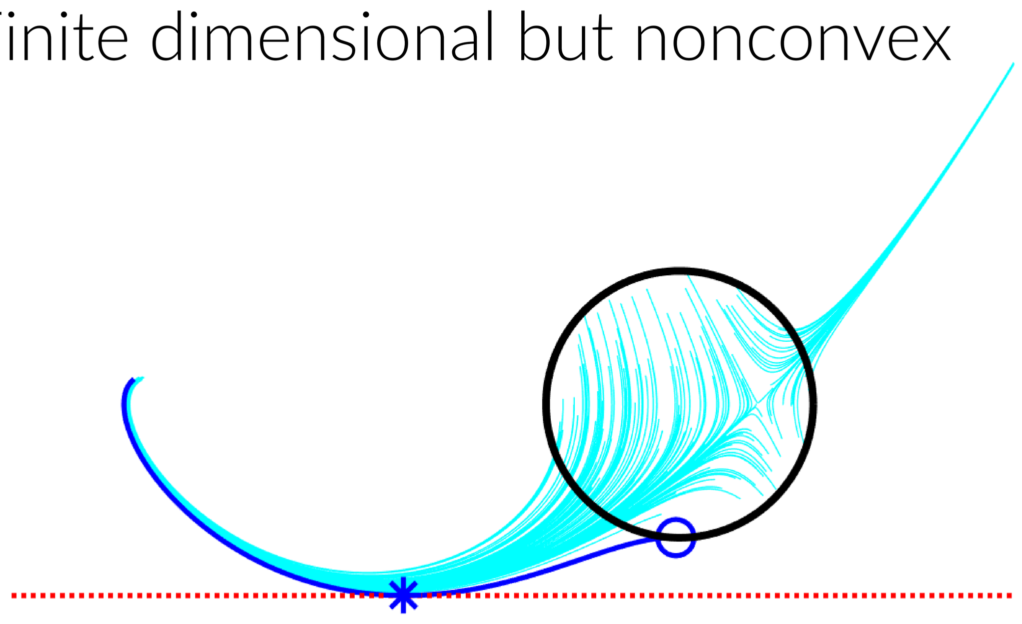
## Peak Estimation

Utilize theory of peak estimation

Find bounds on state function $p(x)$

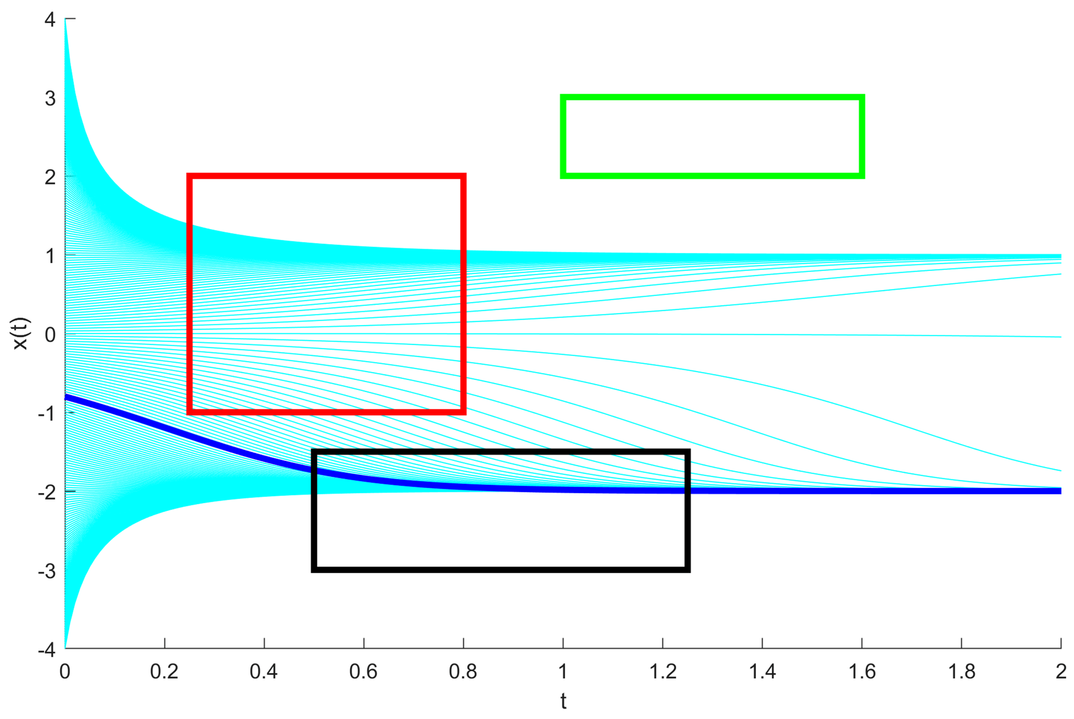
$$P^* = \min_{t,\, x_0 \in X_0} p(x(t \mid x_0))$$
$$\dot{x}(t) = f(t, x(t)) \qquad t \in [0, T]$$

Finite dimensional but nonconvex



## Occupation Measures

Returns the time trajectories spend in each set (given initial distribution)

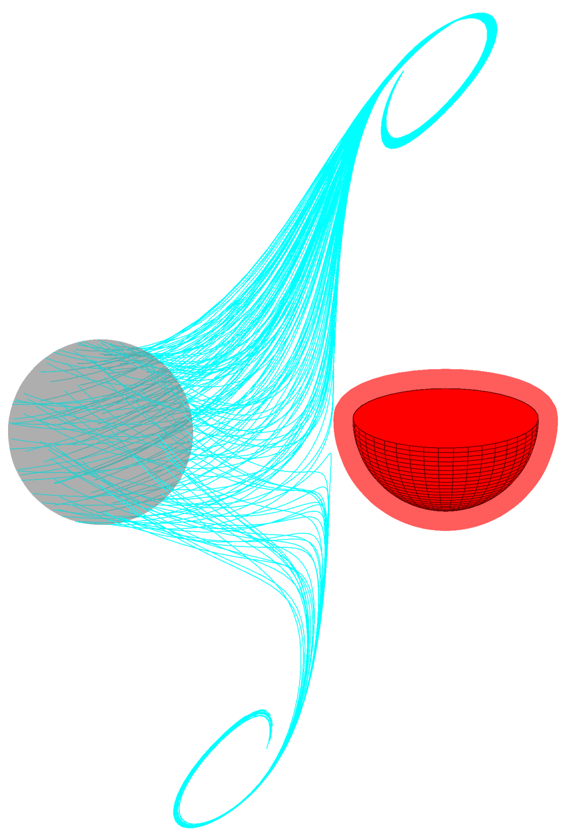



Convex but infinite-dimensional (LP)

## Distance Estimation

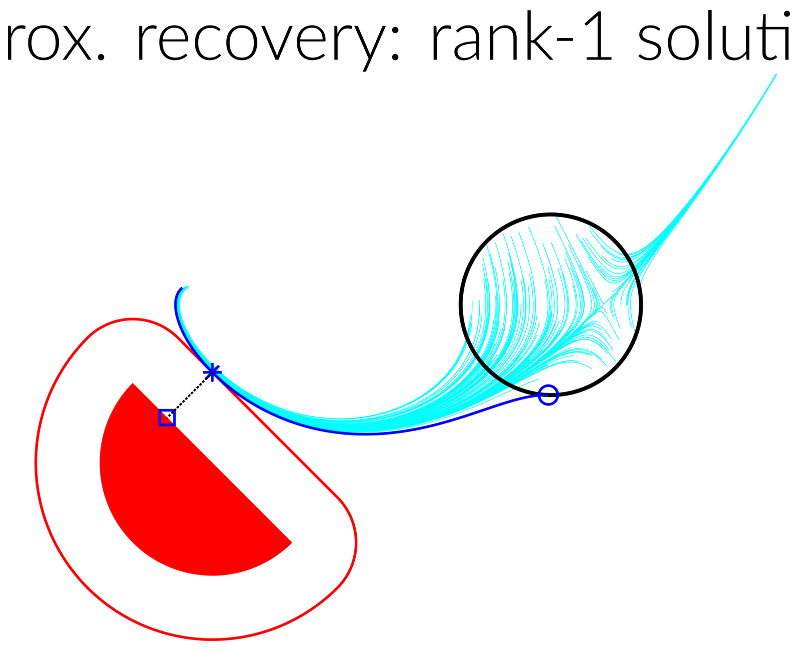
Distance $c(x, y)$ (e.g. Euclidean $\|x - y\|_2$)
Unsafe-set $c(x; X_u) = \min_{y \in X_u} c(x, y)$

Peak estimation with $p(x) = c(x; X_u)$
Variable groups $(t, x)$ and $(x, y)$

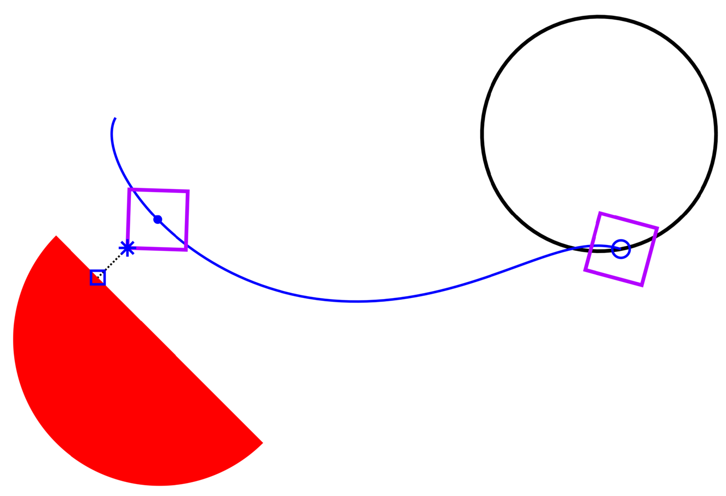


Separability reduces SDP complexity: $c(x, y) = \sum_{k=1}^{n} c_k(x_k, y_k)$

Approx. recovery: rank-1 solutions



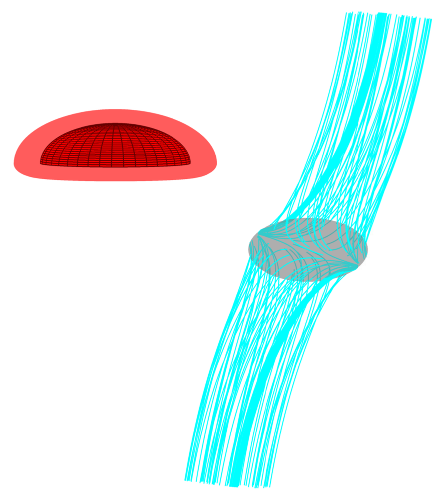
Safety of Points



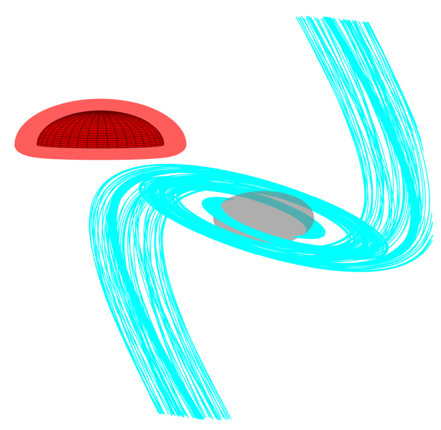
Safety of Shapes (rotating square)

## Hybrid Dynamics

Transitions between locations
Guards and resets (e.g. contact)



Uncontrolled to infinity

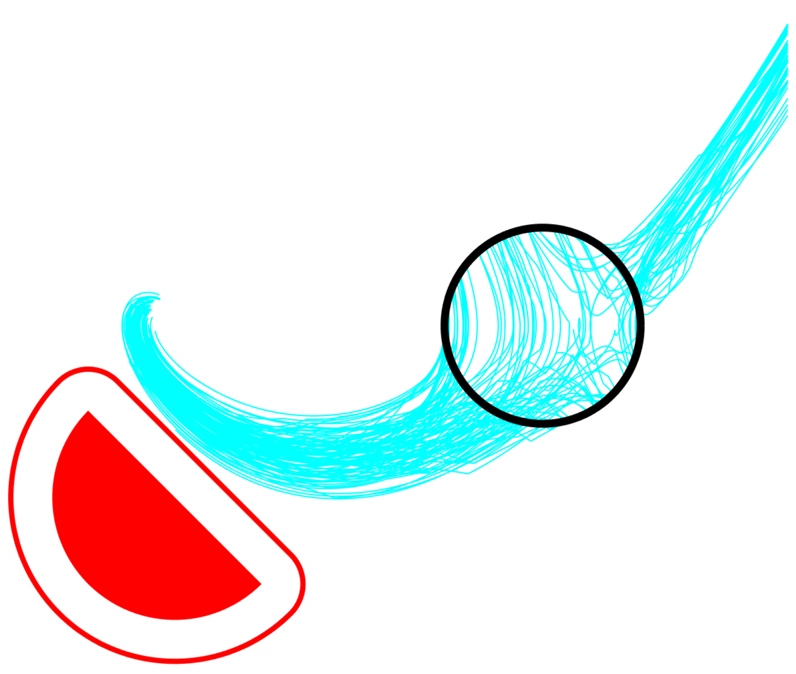


Controlled to gray region

Further details and papers

## Uncertainty

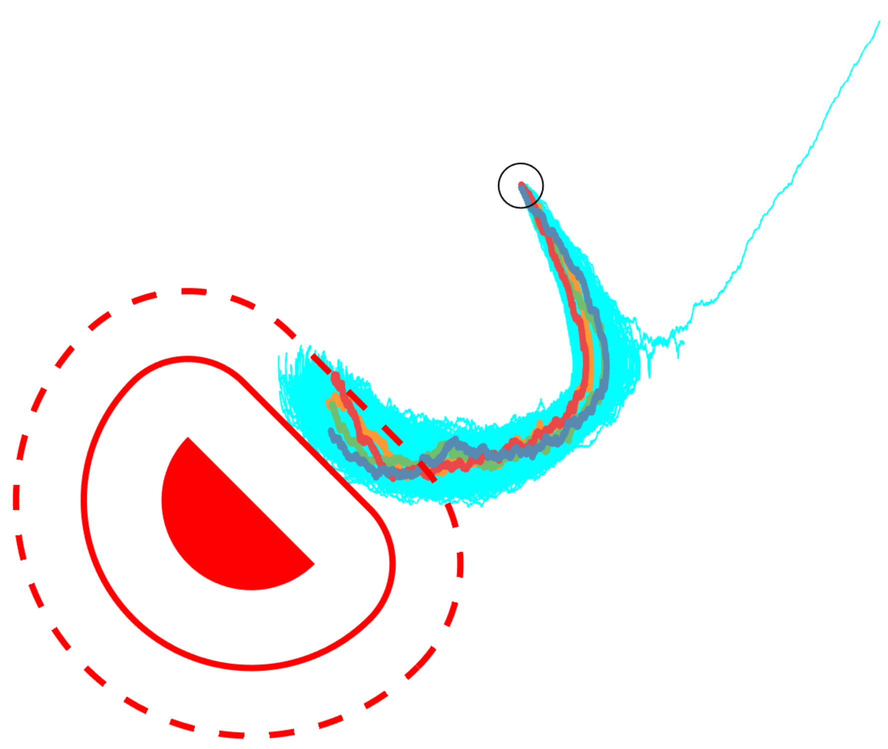
Compactly supported uncertainty
Time-dependent (bounded noise)
or time-independent (parametric)



Exploit polytopic and switching structure

## Stochastic Dynamics

Probabilistic bounds on peak/distance
Use upper-bounds of Value-at-Risk

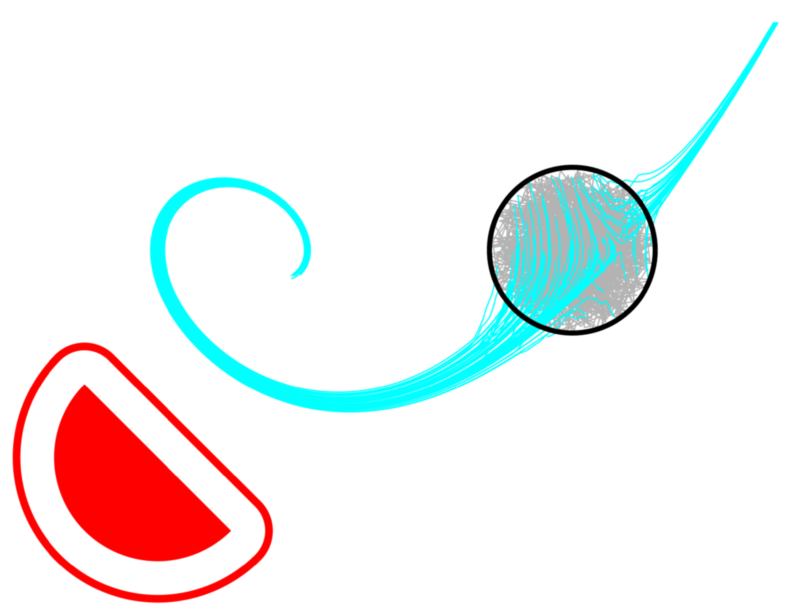


Dotted: 50% bound, Solid: 85% bound

## Time Delay

Discrete time delay $\tau$, history $x_h$

$$\dot{x}(t) = f(t, x(t), x(t - \tau)) \quad t \in [0, T]$$
$$x(s) = x_h(s) \qquad s \in [-\tau, 0]$$



Gray: history, cyan: system trajectory