Symmetry-Reduced Algebraic Selective Harmonic Elimination for Cascaded H-Bridge Converters with Unequal DC Source Levels

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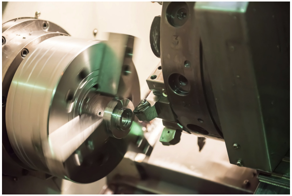
Motivation

Want to efficiently control an electric drive system



Want a nice, clean, sinusoidal current/flux/rotor motion

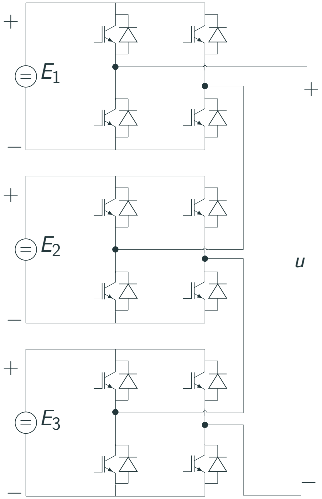
Our problem: DC sources converter are *unequally spaced*

Cascaded H-Bridge Converter

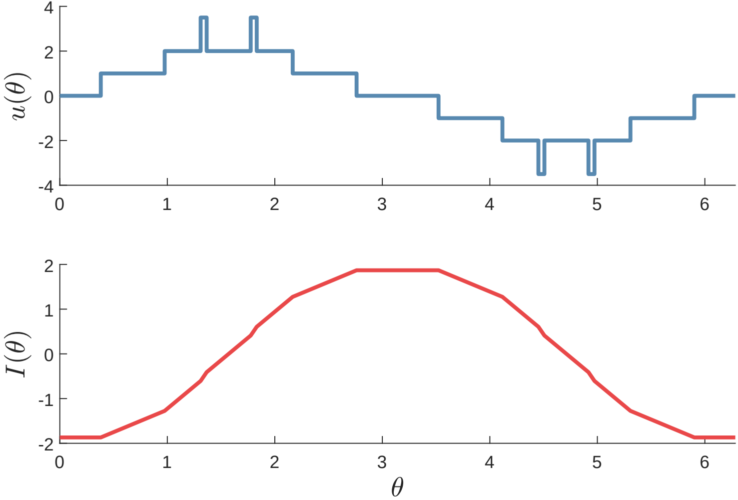
Composed of N_c stacked units

Each H-bridge has switch positions $\sigma_i \in \{-1, 0, 1\}$

DC sources $\{E_i\}_{i=1}^{N_c}$ not all equal, r distinct values $\{\tilde{E}\}$



Pulse Patterns



Voltage u and Current I: steps 2kV, 2kV, 3kV.

Goal

Design a Quarter-and-Half-Wave symmetric pattern u

Sine harmonics $\{b_\ell\}$ should meet specifications

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Must balance between competing objectives (Pareto)

1. Total demand distortion (pure inductance)

$$J_{\mathrm{TDD}} = \sqrt{\sum_{\ell \neq 1,\ 3 \nmid \ell} \frac{b_\ell^2}{\ell^2}}.$$

2. Maximum common-mode voltage

$$J_{\mathrm{cm}} = \max_{\theta \in [0,2\pi]} \tfrac{1}{3} \left| u(\theta) + u(\theta + \tfrac{2\pi}{3}) + u(\theta + \tfrac{4\pi}{3}) \right|,$$

Main Approaches

Find $u(\theta)$ meeting objectives

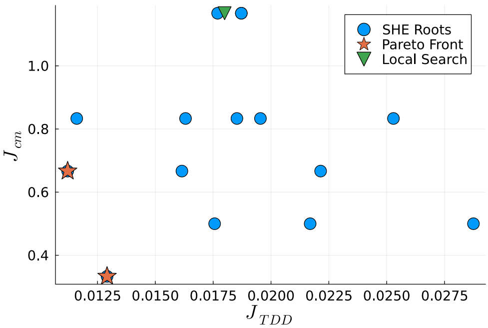
- Carrier-Based/Space Vector Pulse Width Modulation
- **Selective Harmonics Elimination**
- Selective Harmonics *Mitigation* (e.g. IEEE519 code)
- Optimal Pulse Patterns
- (Finite-Set) Model Predictive Control

Method

Selective Harmonics Elimination (SHE) to satisfy constraints

Efficiently compute all SHE-feasible solutions using symmetry

Pick a solution on the Pareto frontier

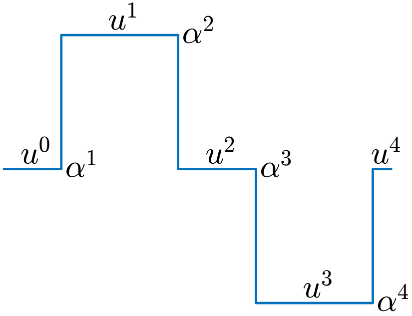


(Unequal) SHE

Anatomy of a Pulse Pattern

Input $u(\theta)$ described by *angles* $\{\alpha^i\}_{i=1}^k$ and *levels* $\{u^i\}_{i=0}^k$

$$u(\theta) = \begin{cases} u^0 & \theta \in [0, \alpha^1) \\ u^i & \theta \in [\alpha^i, \alpha^{i+1}) \\ u^k & \theta \in [\alpha^k, 2\pi) \end{cases}$$



Output voltage level is $u(\theta) = \sum_{j=1}^{N_c} \sigma_j \frac{E_j}{2}$

Unequal SHE System

Constraint d low-order differential-mode harmonics:

$$H \subset (1, 5, 7, 11, 13, \ldots), \qquad |H| = d$$

Desired amplitude M of fundamental component

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Harmonics constraints: system of nonlinear equations

$$\begin{aligned} b_1 &= \textstyle\sum_{i=1}^{d} -\Delta u^i \cos(\alpha_i) = M\frac{\pi}{4} \\ b_\ell &= \textstyle\sum_{i=1}^{d} -\Delta u^i \, T_\ell(\cos(\alpha_i)) = 0 \qquad \forall \ell \in H \setminus 1, \end{aligned}$$

Chebyshev polynomials T_n, Voltage gaps $\Delta u^i = u^i - u^{i+1}$

Partitioning

d switches/quarter period, r unique voltage values E

$\binom{d+r-1}{d}$ partitions of the k switches into r patterns

Solve an SHE problem for each partition γ [1]

$$\sum_{j=1}^{r} \frac{\tilde{E}_j}{2} \left(\sum_{\nu=1}^{\gamma_j} c_{j\nu} \right) = M \frac{\pi}{4}$$

$$\sum_{j=1}^{r} \frac{\tilde{E}_j}{2} \left(\sum_{\nu=1}^{\gamma_j} T_\ell(c_{j\nu}) \right) = 0 \qquad \forall \ell \in H \setminus 1.$$

[1] K. Yang, X. Tang, Q. Zhang, and W. Yu, "Unified selective harmonic elimination for fundamental frequency modulated multilevel converter with unequal DC levels," in IECON 2016-42nd Annual Conference of the IEEE Industrial Electronics Society. IEEE, 2016, pp. 3623–3628.

Symmetry Reduction

SHE is invariant under permutations within same level

Mass duplication of roots, computationally wasteful

Use elementary symmetric polynomials to reduce SHE [2]

$$
\begin{aligned}
e_{j0}(x) &= 1, \qquad e_{j\gamma_j} = \prod_{\nu=1}^{\gamma_j} c_{j\nu}, \\
e_{j\nu}(x) &= \sum_{\beta_1<\beta_2<\ldots<\beta_\nu} c_{j\beta_1} c_{j\beta_2} \ldots c_{\beta_n u} \qquad 1 \le \nu < \gamma_j, \\
e_{j\nu}(x) &= 0 \qquad \nu > \gamma_j.
\end{aligned}
$$

[2] K. Yang, Q. Zhang, R. Yuan, W. Yu, J. Yuan, and J. Wang, "Selective harmonic elimination with Groebner bases and symmetric polynomials," IEEE Trans. Power Electron., vol. 31, no. 4, pp. 2742–2752, 2015.

Symmetry Reduction (cont.)

Unequal SHE converted into *normal form* ($\exists$ unique $g(e)$) :

$$\sum_{j=1}^{r} \frac{\tilde{E}_j}{2} e_{j1} = M\frac{\pi}{4}, \qquad \forall \ell \in H \setminus 1 : \sum_{j=1}^{r} g_{j,\ell}(e_j) = 0$$

Preprocessing by Newton-Girard Identities, Groebner bases [3]

Solve for e by Homotopy Continuation

[3] C. Wang, Q. Zhang, D. Chen, Z. Li, W. Yu, and K. Yang, "Application of Newton Identities in Solving Selective Harmonic Elimination Problem With Algebraic Algorithms," IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 10, no. 5, pp. 5870–5881, 2022.

Recover Roots

Angle cosines c extracted from symmetry roots e

$$c_j = \{t \mid t^{\gamma_j} + t^{\gamma_j - 1} e^*_{j1} + t^{\gamma_j - 2} e^*_{j2} + \ldots + t e^*_{j\ \gamma_j - 1} + e^*_{j\gamma_j} = 0\}.$$

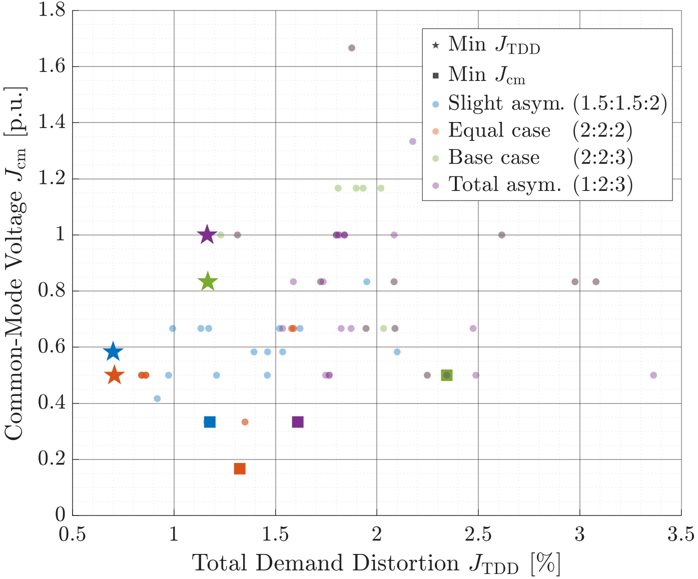
Recover switch positions and angles: Universal formulation

$$(\sigma_{j\nu}, \alpha_{j\nu}) := \begin{cases} (1, \cos^{-1}(c^*_{j\nu})) & c^*_{j\nu} \geq 0 \\ (-1, \pi - \cos^{-1}(c^*_{j\nu})) & c^*_{j\nu} < 0. \end{cases}$$

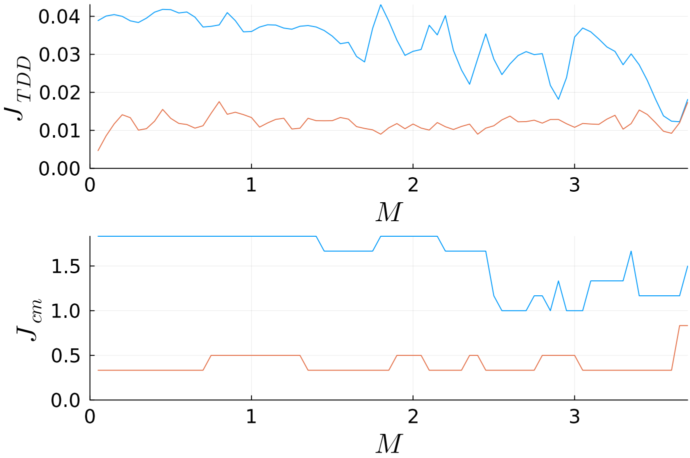
Restrict to physically implementable patterns by CHB

Examples

Three-cell H-bridge (M=2)



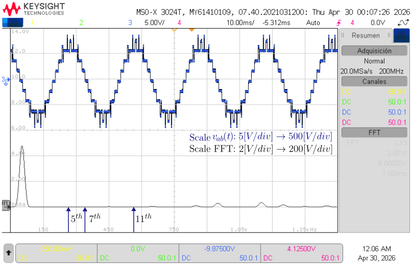
Three-cell H-bridge (swept M)



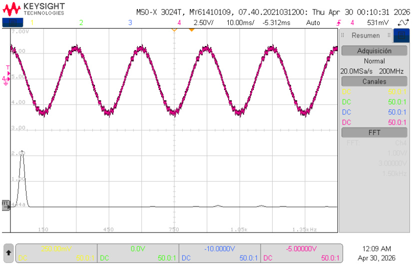
Non-monotonic SHE curves: $E = (2, 2, 3)$, $d = 4$

HITL Validation

$E = (2, 2, 3), M = 2$, minimum TDD pattern (1.167 %)



Voltage

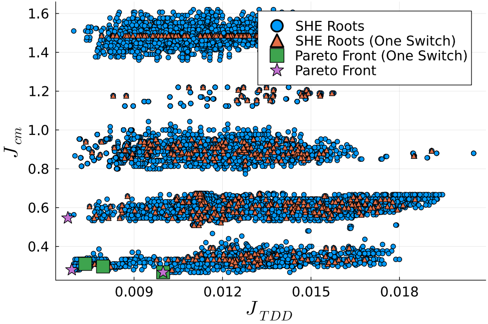


Current

Suppression of harmonics (5, 7, 11)

Five-cell H-Bridge

Distinct[4] $E = (1, 0.93, 0.89, 0.84, 0.8),\ M = 0.8\frac{4}{\pi}, d = 5$



7921 SHE patterns, 592 have one switch/cell/quarter period

[4] K. Yang, Q. Zhang, J. Zhang, R. Yuan, Q. Guan, W. Yu, and J. Wang, "Unified selective harmonic elimination for multilevel converters," IEEE Transactions on Power Electronics, vol. 32, no. 2, pp. 1579–1590, 2016

Take-aways

Conclusion

Finding unequal SHE roots is difficult

Symmetry reduction improves computational tractability

Pick best solution from the Pareto frontier

Efficient Power!