Are Optimal Pulse Patterns Optimal?

Bounding harmonic distortion in power converters

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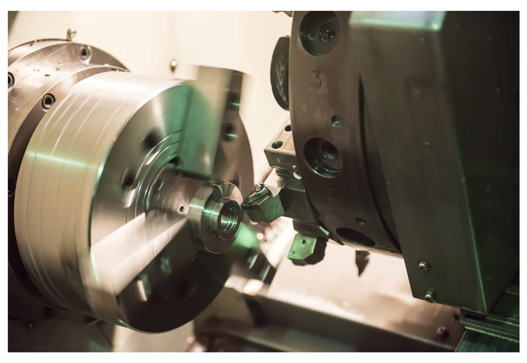
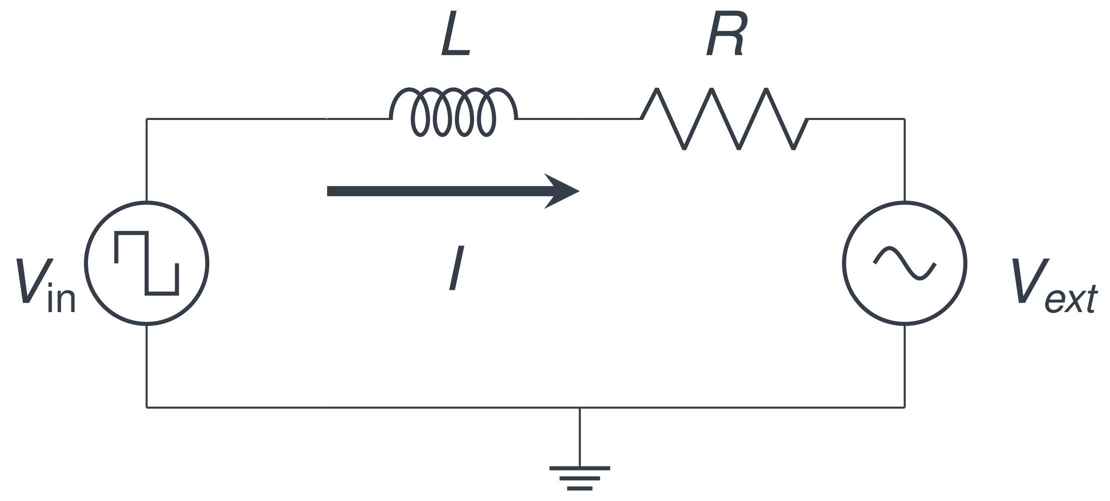
Motivation

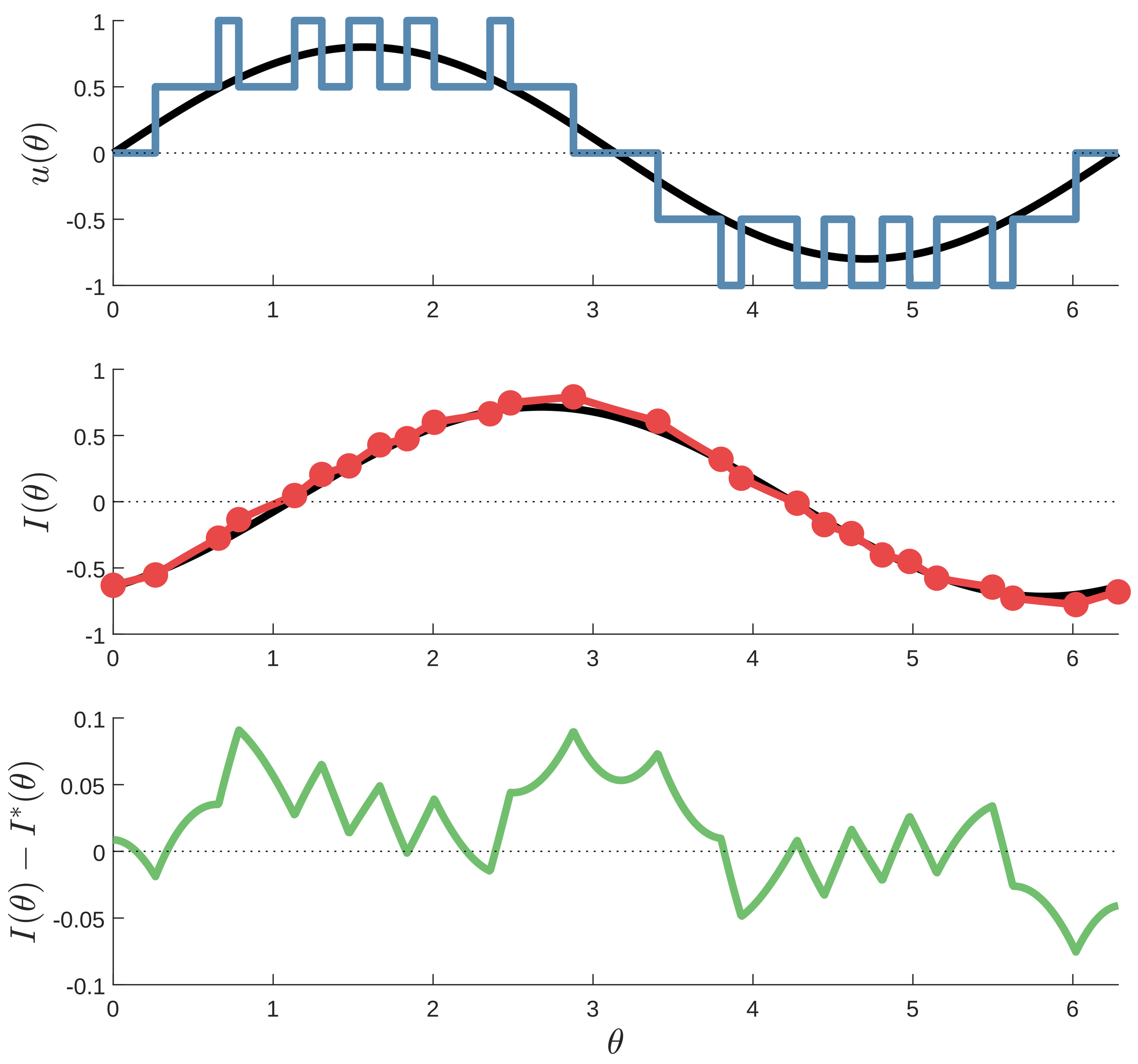
Electric drive systems are a core technology in industry and modern life.


These systems must be strictly controlled in order to produce desired motor speed, torque, current, and magnetic flux responses.



DC/AC power converters (inverters) are a component in motor control. Inverters can switch between a finite number of voltage levels to produce a load current I. The goal of this work is to bound the minimum possible energy draw $||I||_2^2$, which is monotonically related to the tracking error $||I - I^*||_2^2$ and the current demand distortion of I.



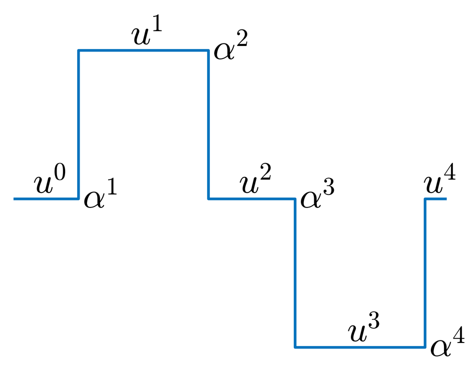
Suboptimality 2.151×10^{-5} in $||I||_2^2$, with 32 switches and $R/L = 0.5$

Minimizing these objectives improves energy efficiency, increases component lifetime, maintains power quality, and reduces economic costs.

Optimal Pulse Patterns

This minimization is an instance of a *nonconvex* Optimal Pulse Pattern problem [1]: choose a sequence $u(\theta)$ with levels u^i and angles α^i:

$$u(\theta) = \begin{cases} u^0 & \theta \in [0, \alpha^1) \\ u^i & \theta \in [\alpha^i, \alpha^{i+1}) \\ u^k & \theta \in [\alpha^k, 2\pi) \end{cases}$$



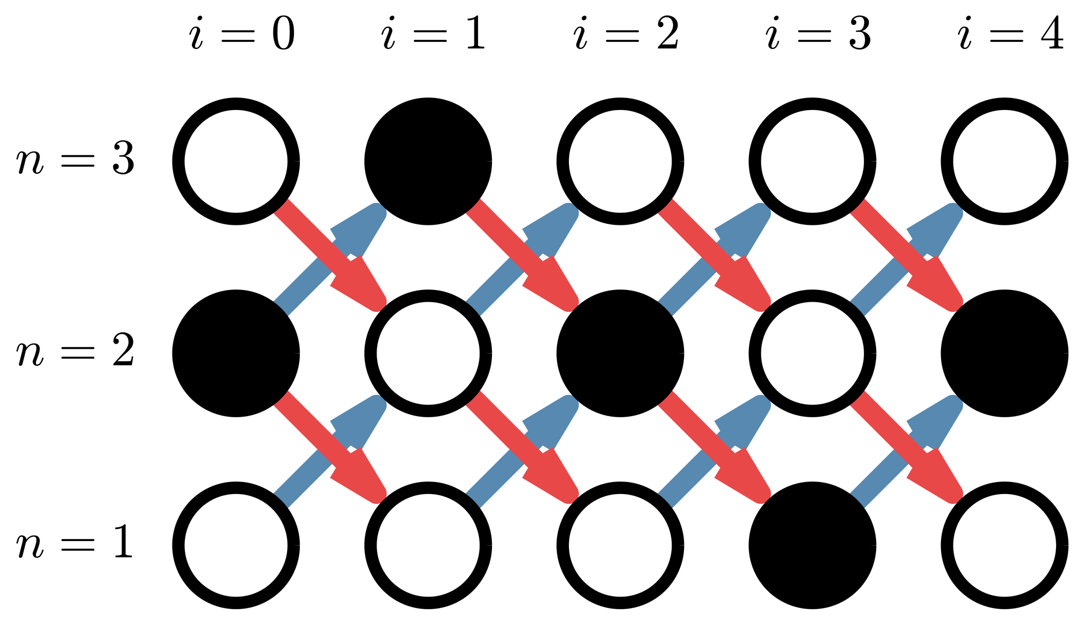
Possible (nonconvex $\star$) constraints/considerations include

- Switching Frequency
- Spacing Between Switches
- Harmonics Specifications $\star$
- Symmetries
- Switching Restrictions $\star$
- Power Budget $\star$

Other methods for pattern design include Selective Harmonics Elimination, Carrier/Space Vector Modulation, Finite-set MPC, and MP^{3}C .

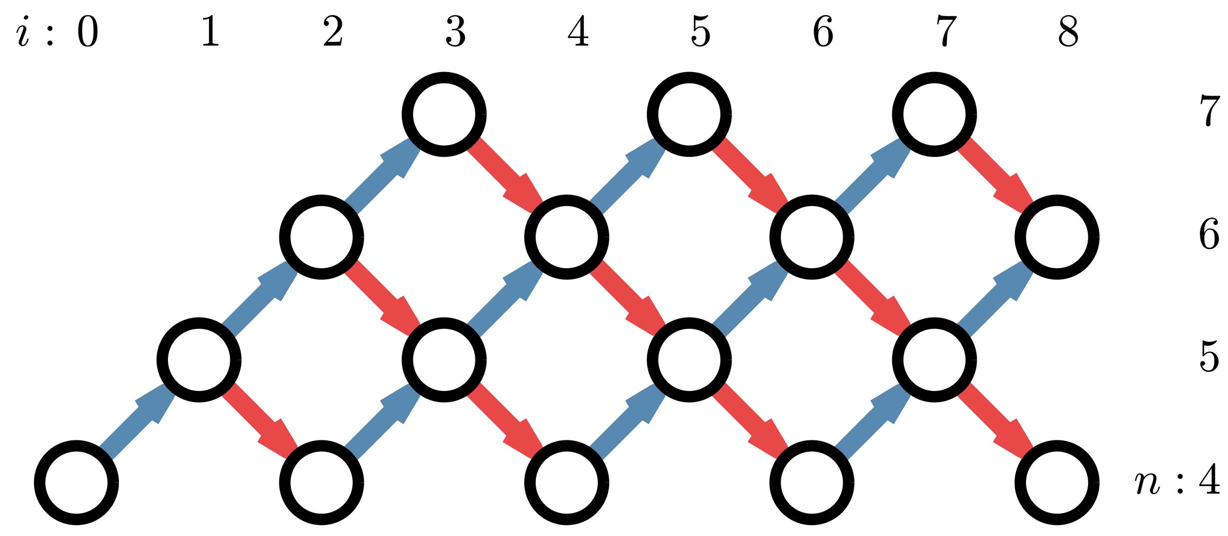
Optimal Pulse Patterns as Optimal Control

We convert Optimal Pulse Patterns into a mode-selecting periodic Optimal Control Problem in a hybrid system. The modulation levels $\{u^i\}$ are converted into a periodic path through a transition graph



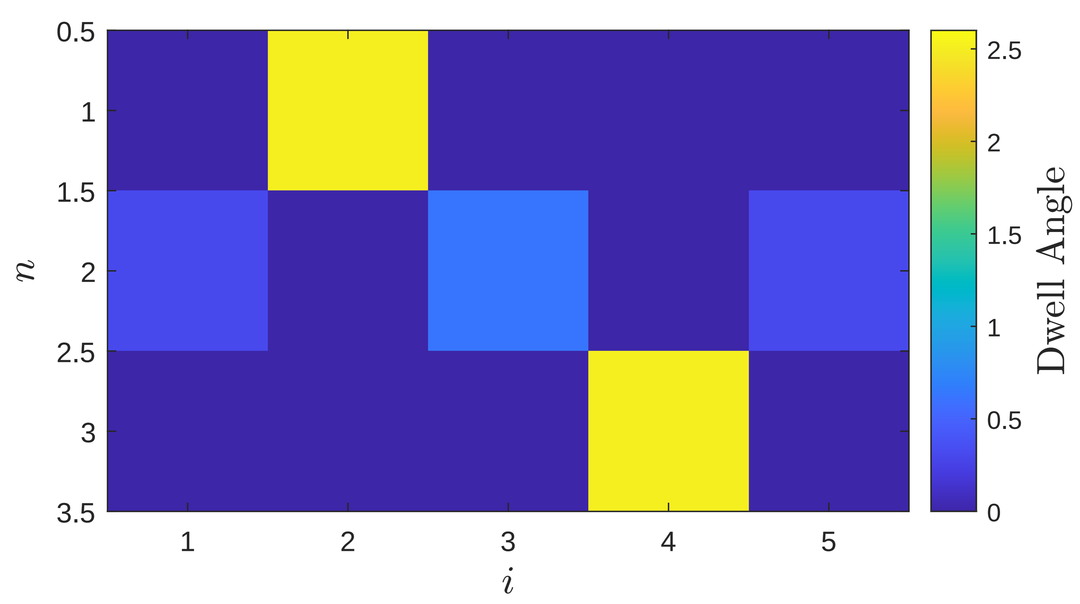
Path representation

Symmetry and restricted switching can be implemented by graph pruning



Unipolar and Quarter-Wave symmetry (for $\theta \in [0, \pi/2]$)

The angle differences $\alpha^{i+1} - \alpha^i$ correspond to the time taken at a mode



Dwell/Occupancy table representation

The tracked dynamics involve the angle θ, clock angle since last jump ϕ, and load current I_L. Per-mode and jump dynamics within $\theta \in [0, 2\pi]$ are

when in mode: $\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ u^i - \frac{R}{L} I \end{bmatrix}$ upon leaving mode: $\begin{bmatrix} \theta^+ \\ \phi^+ \\ I_L^+ \end{bmatrix} = \begin{bmatrix} \theta \\ 0 \\ I \end{bmatrix}$

The mode-selecting optimal control reformulation remains nonconvex. We then use existing convex relaxation techniques in optimal control to lower-bound the energy cost [2]. Solving a sequence of semidefinite programs in rising polynomial degree gives increasingly accurate lower bounds to the minimal energy draw in a single-phase electric drive. Future work will include three-phase considerations.

Cooperations

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- Tobias Geyer (ABB Industrial Drives, Switzerland)

References

[1] Giuseppe S Buja.
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[2] Pengcheng Zhao, Shankar Mohan, and Ram Vasudevan.
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