Exploiting SDP Structure Yields Tighter Approximations

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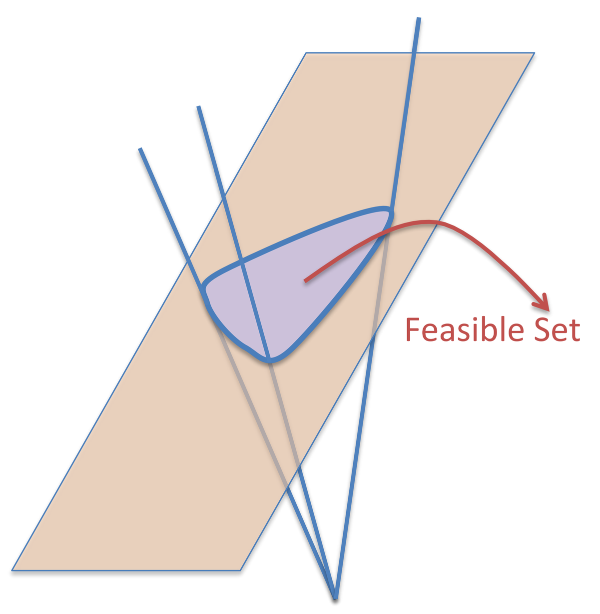
Semidefinite Programs

Modern control problems require solutions of large scale SDPs



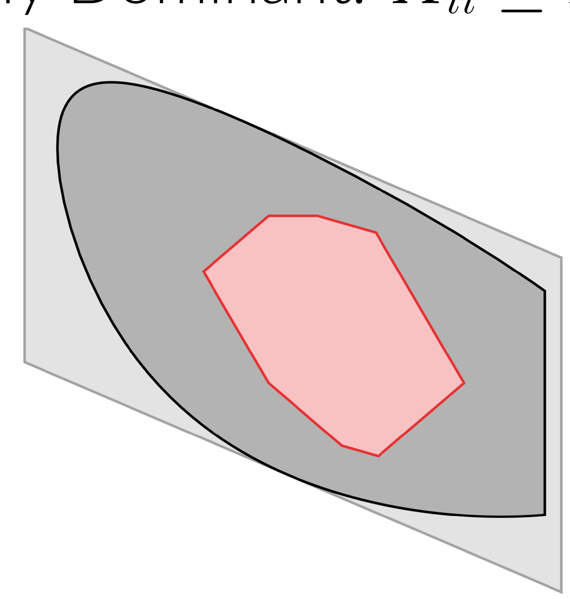
$$\min_X \langle C, X\rangle$$
$$b_i = \langle A_i, X\rangle \,|_{i=1}^m$$
$$X \in \mathbb{S}_+^n$$

Runtime scales as $O(n^2m^2 + n^3m)$

SDP Approximation

Structured (simpler) subset of PSD

Diagonally Dominant: $X_{ii} \geq \Sigma_{i\neq j}|X_{ij}|$



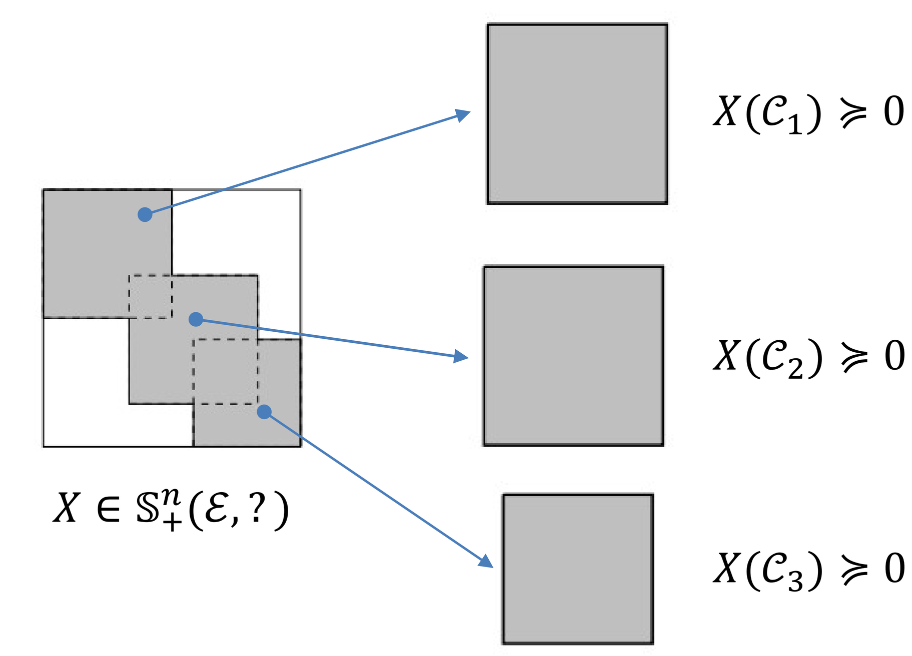
$\mathcal{DD} \subset \mathbb{S}_+ \subset \mathcal{DD}^*$

$\mathcal{DD}$: Upper and Lower bounds by LP

SDP Structure

Improve runtime by reducing n

Decompose based on structure



Includes sparsity, symmetry, *-algebra

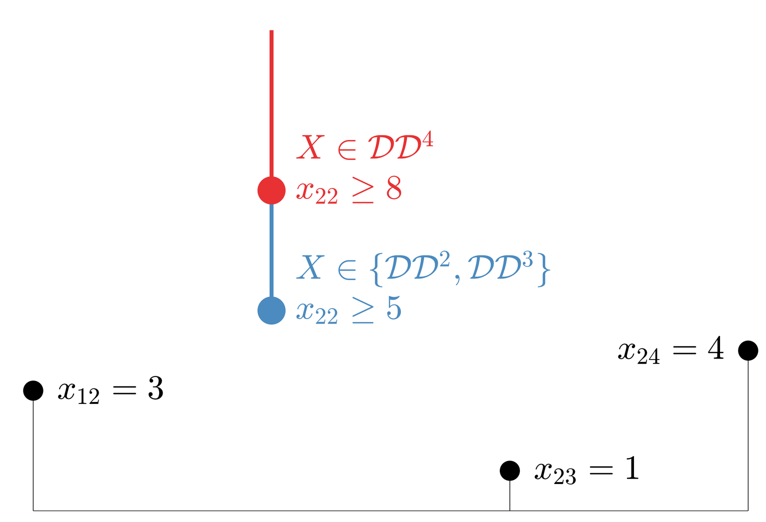
Structure Broadens Feasible Regions

Approximations destroy structure, worse runtime and bounds

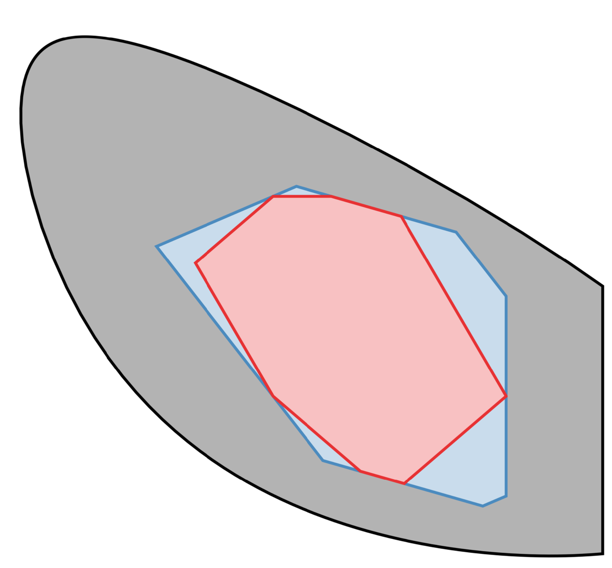
$$\begin{pmatrix} x_{11} & x_{12} & ? & ? \\ x_{12} & x_{22} & x_{23} & x_{24} \\ ? & x_{23} & x_{33} & x_{34} \\ ? & x_{24} & x_{34} & x_{44} \end{pmatrix} \in \mathbb{S}_+^4$$

Cliques are $\{(1,2), (2,3,4)\}$

Matrix is $\mathcal{DD}$ vs. Cliques are $\mathcal{DD}$



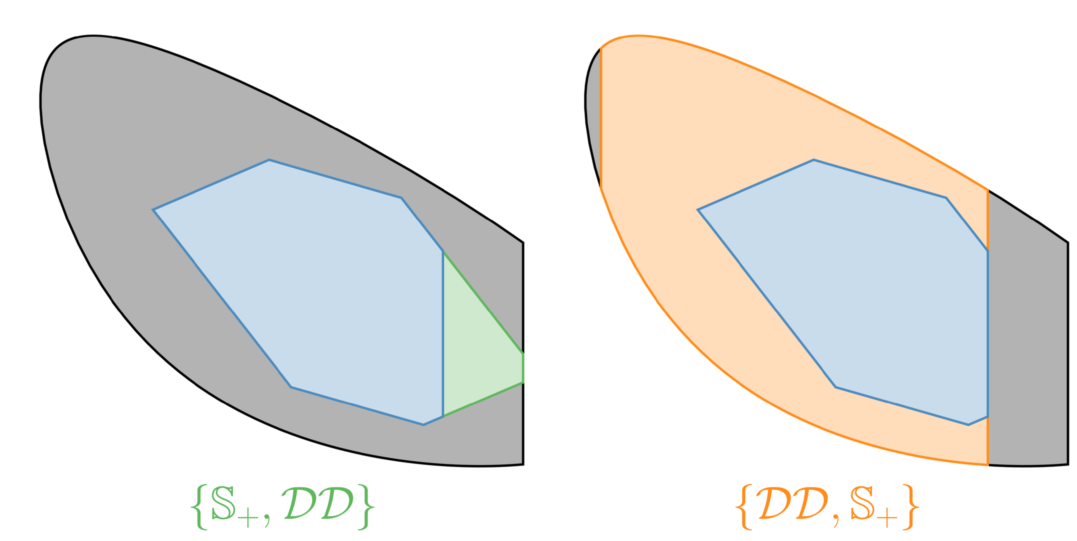
Minimal value of x_{22}



$\mathcal{DD} \subset \{\mathcal{DD}, \mathcal{DD}\} \subset \mathbb{S}_+$

Mixing Cones

Adds flexibility in optimization



Useful if problem has few large cliques

Example: Polynomial Optimization

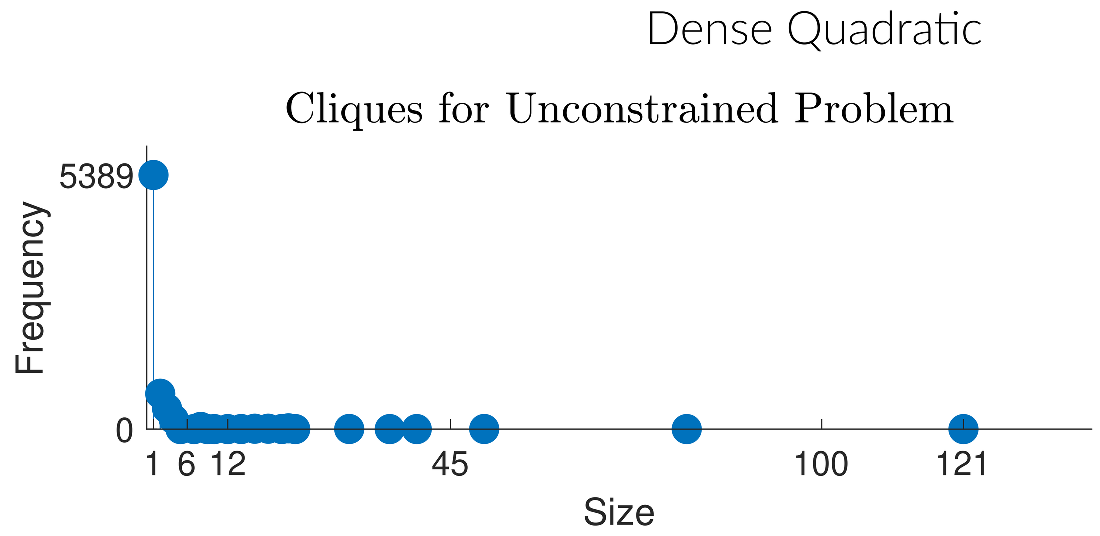
Lower bounds by 2nd order Moment-SOS, decomposition by term sparsity

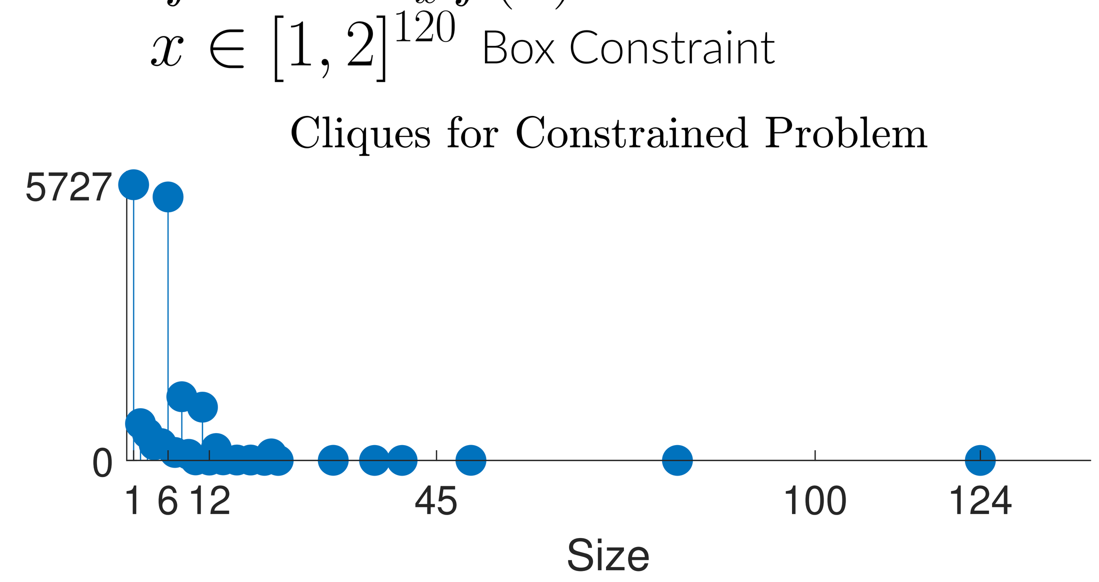
Sparse Quartic

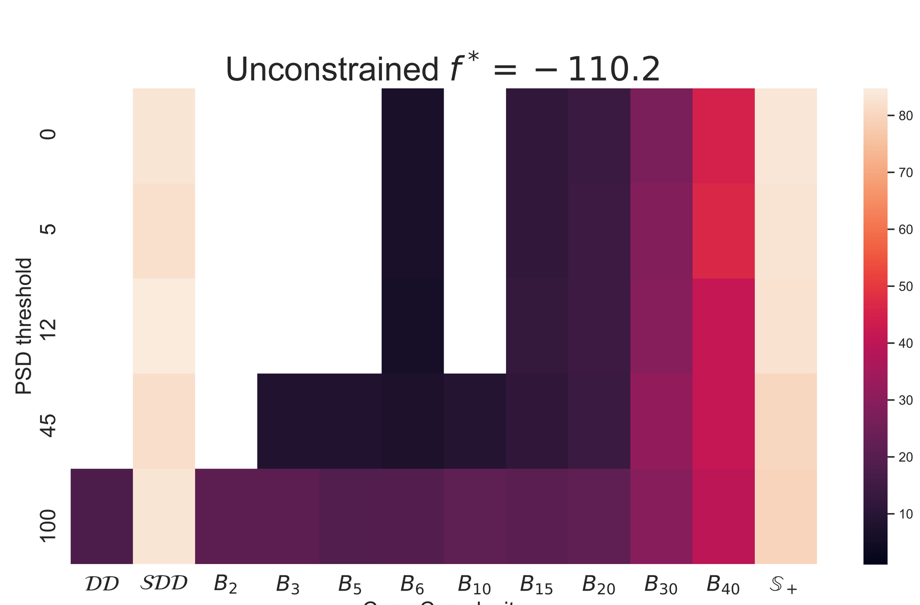
$$f(x) = f_R(x) + f_Q(x)$$

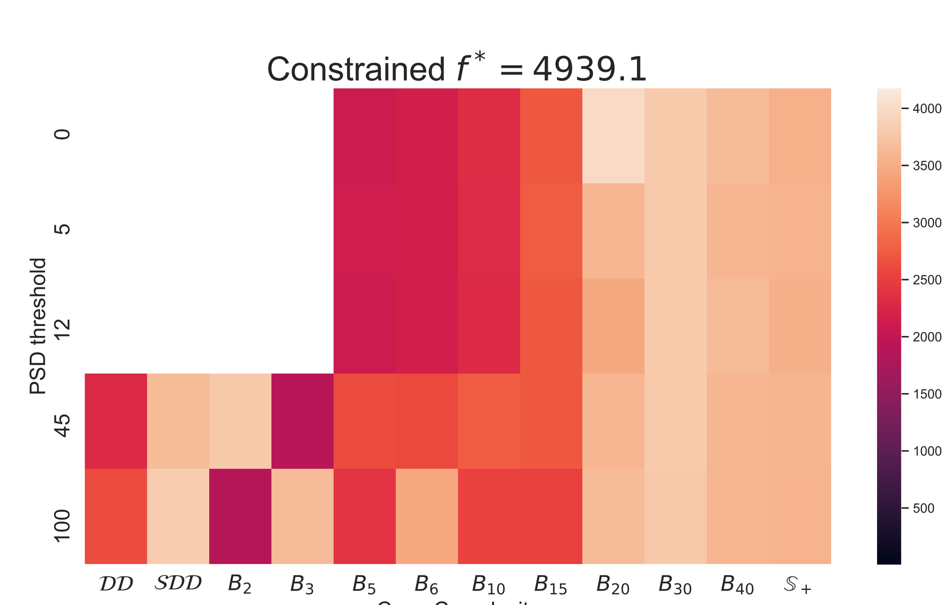
Dense Quadratic

$$f^* = \min_x f(x)$$
$$x \in [1,2]^{120}$$ Box Constraint







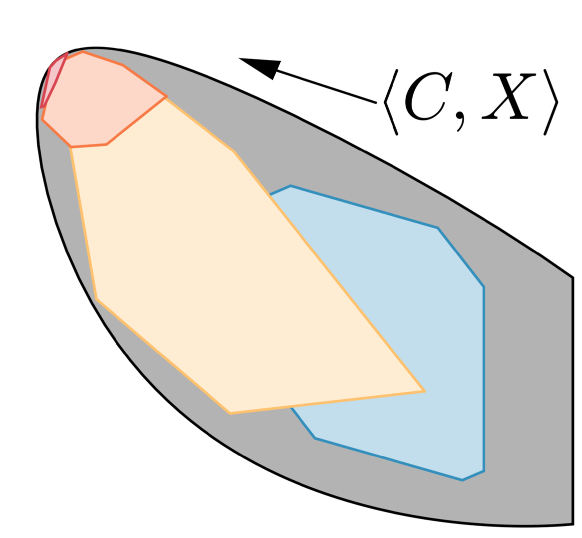


Time to find SDP-matching lower bounds (seconds)

Implications

Structure improves approximations

Change of Basis: iterative refinement



Maximize cost: $p_0 \leq p_1 \leq p_2 \leq p_3$

Future steps:

- Convergence to SDP optimum
- Optimal Power Flow
- H_2/H_∞ Network Control

arXiv:1911.12859

github.com/zhengy09/SDPfw