A Model of Heave Dynamics for Bagged Air Cushioned Vehicles

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Air Cushioned Vehicles

Air Cushioned Vehicle

Crafts supported by cushion of pressurized air

Robust craft, can handle waves and rough terrain

Dynamics principles formulated by Sir Christopher Cockerell in 1955

Figure 1: US Navy LCAC

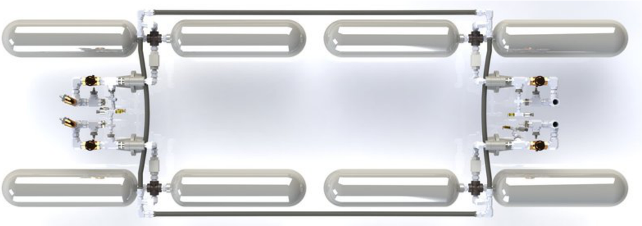
Paradigm Hyperloop

Team for SpaceX Hyperloop Competition (MUN, NEU, CNA)

Figure 2: Competition 2 Pod

Subsystems

(a) Levitation and Suspension subsystem ($M = 909$ kg)



(b) Air supply subsystem ($\dot{m}_{in} = 0.16$ kg/s)

Pod in Atmosphere (101.3 kPa)

Pod Video Atmospheric Pressure

Pod at Tube Pressure (0.66 kPa)

Pod Video Tube Pressure

Motivation

Create full dynamic model for pod motion

Table 1: 6DOF of Pod motions

Translational		Rotational	
Heave	Focus of Paper	Roll	Out of Scope (3d)
Surge	External Propulsion	Pitch	Out of Scope (3d)
Sway	Spring/Mass Damper	Yaw	Spring Mass Damper

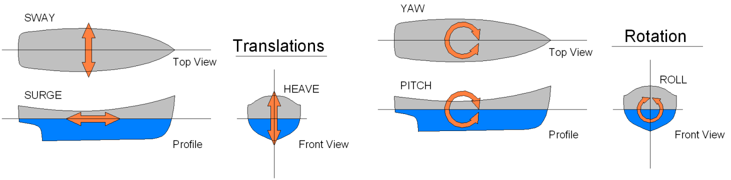


Figure 4: Credit to Jmvolc on Wikipedia 'Ship Motions'

System Model

Sowayan and Khalid Model (2013)

3 state open-skirted ODE model

- z: Craft Height
- $\dot{z}$: Vertical Speed
- p: Chamber Pressure

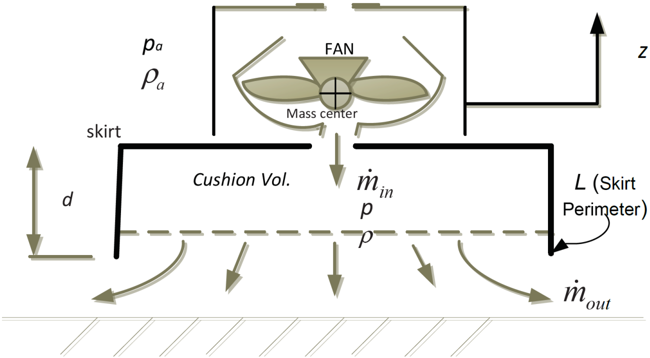


Figure 5: Image from [1]

Model Equations

Dynamics:

$$M\ddot{z} = (p - p_{atmo})A - Mg \tag{1a}$$

$$\dot{p} = \frac{\gamma RT}{A(z+d)}\left(\dot{m}_{in} - \dot{m}_{escape} - \frac{pA\dot{z}}{RT}\right) \tag{1b}$$

Arithmetic Expressions:

$$\dot{m}_{escape} = \frac{c_0 pLz}{\sqrt{RT}}\sqrt{\frac{2\gamma}{\gamma-1}\left(\left(\frac{p_{atmo}}{p}\right)^{\frac{2}{\gamma}} - \left(\frac{p_{atmo}}{p}\right)^{\frac{\gamma+1}{\gamma}}\right)} \tag{1c}$$

Based on Bernoulli's Equation and Isentropic Expansion

Pod 2 in Atmosphere

- $\dot{m}_{in}^{*} = 0.16$ kg/s, $z^{*} = 0.12$ mm
- Equilibrium is unstable

Bagged Model

Add porous bag to retain pressurized air (reduce $\dot{m}_{escape}$)

Darcy (linear, [2]):

$$\dot{m}_{escape} \approx -\frac{\rho_{atmo}\kappa A_e}{\mu}\frac{\partial P}{\partial z} \tag{2a}$$

Forchheimer (quadratic, [3]):

$$\rho_{atmo}\frac{\partial P}{\partial z} \approx \beta(\frac{\dot{m}_{escape}}{A_e})^2 + \frac{\mu}{\kappa}(\frac{\dot{m}_{escape}}{A_e}) \tag{2b}$$

Escape area $A_e = Lz$

Equations of Motion

Dynamics:

$$M\ddot{z} = (p - p_{atmo})A - Mg \tag{3a}$$

$$\dot{p} = \frac{\gamma RT}{Az}(\dot{m}_{in} - \dot{m}_{escape} - \frac{pA\dot{z}}{RT}) \tag{3b}$$

Arithmetic Expressions:

$$\rho_{atmo}\frac{\partial P}{\partial z} = \beta(\frac{\dot{m}_{escape}}{A_e})^2 + \frac{\mu}{\kappa}(\frac{\dot{m}_{escape}}{A_e}) \tag{3c}$$

$$A_e = Lz \tag{3d}$$

$$\frac{\partial P}{\partial z} = \frac{p - p_{atmo}}{\ell} \tag{3e}$$

Darcy/Forchheimer Comparision (Linearization)

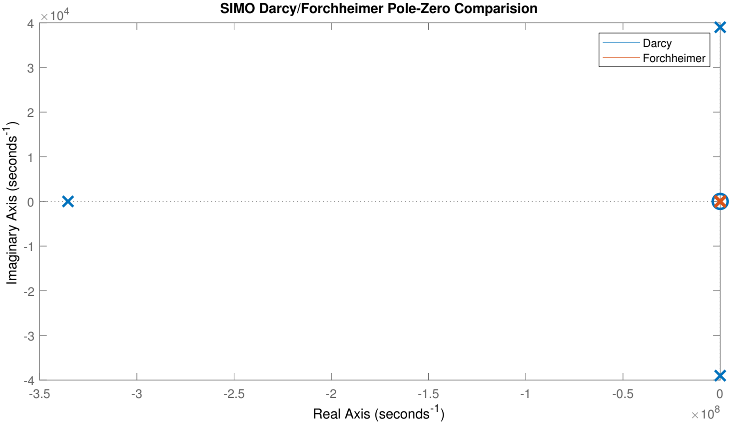
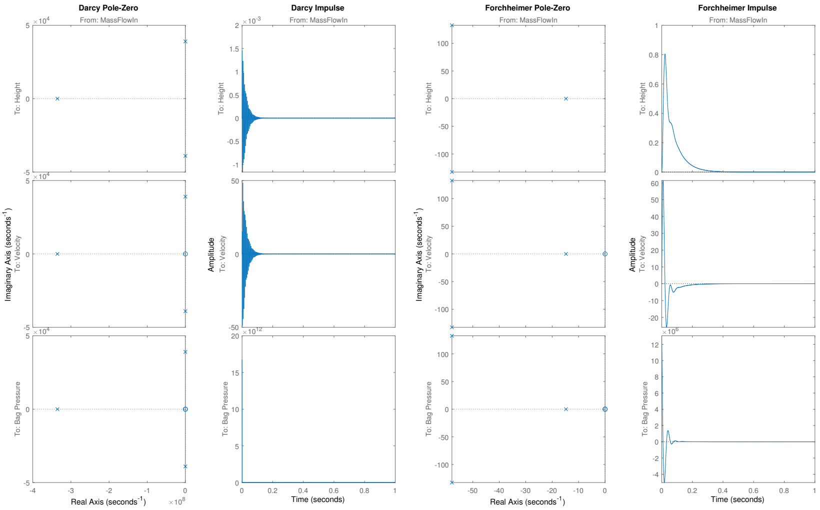


Figure 6: Forchheimer pulls leftmost pole from $-3.35 * 10^8$ to -14.8 rad/s

$\dot{m}_{in} = 0.16$ kg/s, $z^*_{\text{darcy}} = 6.5$ nm, $z^*_{\text{forch}} = 0.83$ mm

Darcy/Forchheimer (with Impulse Response)



Open Loop Stability in Atmosphere

Bilinear Sum-of-Squares optimization [4]

Dynamics Taylor-expanded to 5th order

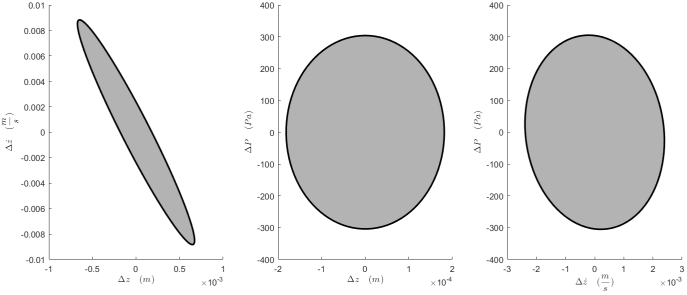
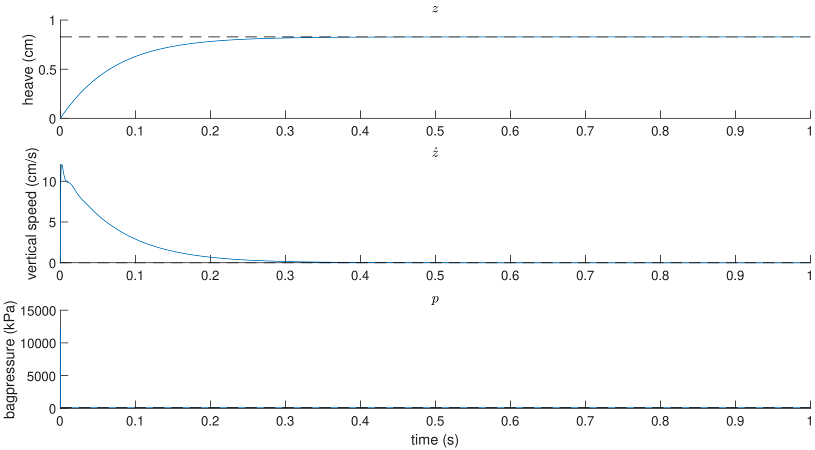


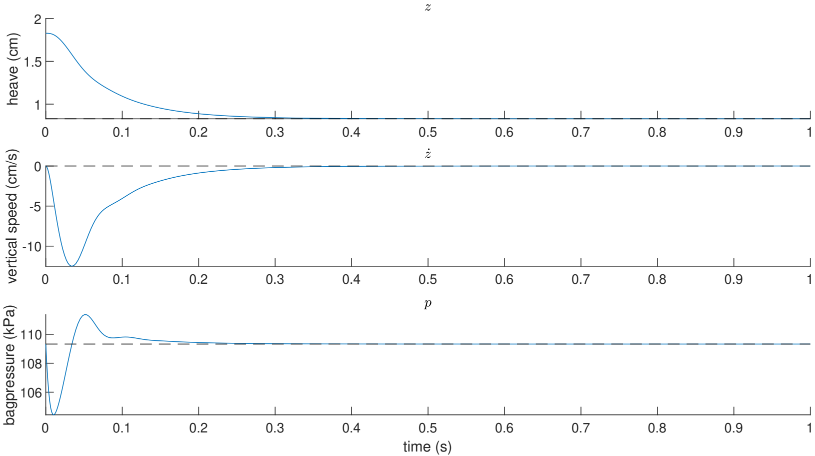
Figure 7: 1-sublevel set of $V(x)$

Dynamics Simulation

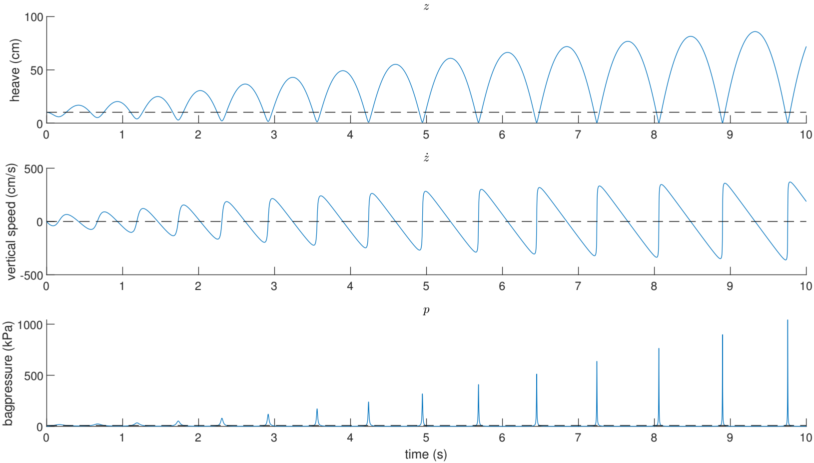
Pod Liftoff (atmosphere)



Pod Drop (atmosphere)



Pod Drop (tube)



Closed Loop System

Flow controllers are $G\frac{3}{8}$ pneumatic MPYE's from FESTO

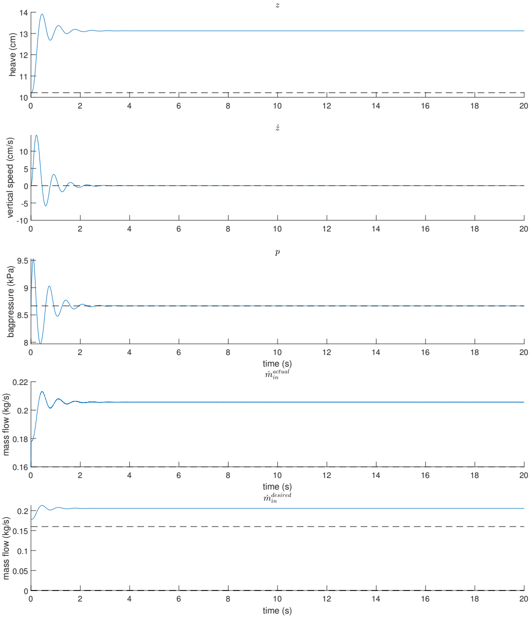
Have critical frequency of 65 Hz $= 408.4$ rad/s

Add new state $\dot{m}_{in}$ to system:

$$\ddot{m}_{in} = 408.4(\dot{m}_{in}^{\text{desired}} - \dot{m}_{in}) \tag{4}$$

Figure 8: Example of MPYE flow controller

Pod Drop LQR Control (tube)

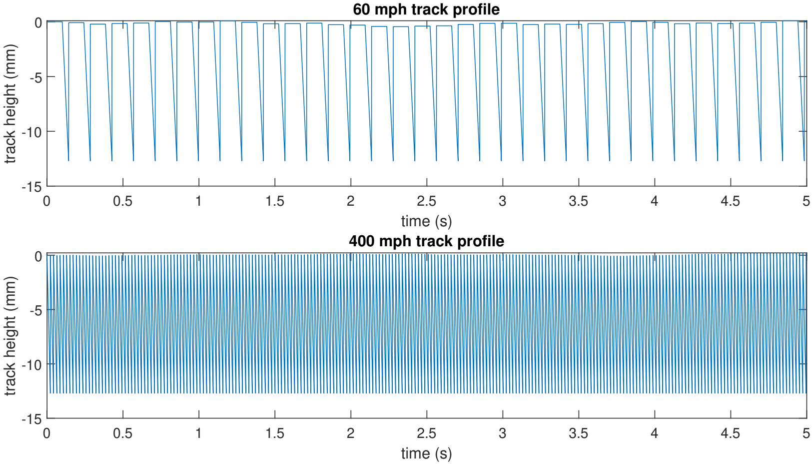


External Disturbances

Track Profile

Subtrack made of 12.5' plates

0.25" horizontal gap, ± 0.004" vertical drift



Disturbed System

Dynamics:

$$M\ddot{z} = (P_{bag} - P_{atmo})A - Mg \tag{5a}$$

$$\dot{p} = \frac{\gamma RT}{A(z + w_{height})}(\dot{m}_{in} - \dot{m}_{escape} - \frac{P_{bag}A\dot{z}}{RT}) \tag{5b}$$

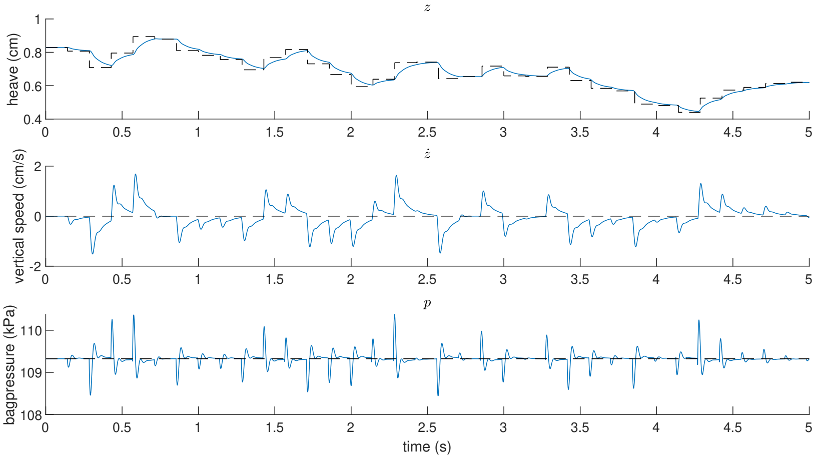
Arithmetic Expressions:

$$\rho_{atmo}\frac{\partial P}{\partial z} = \beta(\frac{\dot{m}_{escape}}{A_e})^2 + \frac{\mu}{\kappa}(\frac{\dot{m}_{escape}}{A_e}) \tag{5c}$$

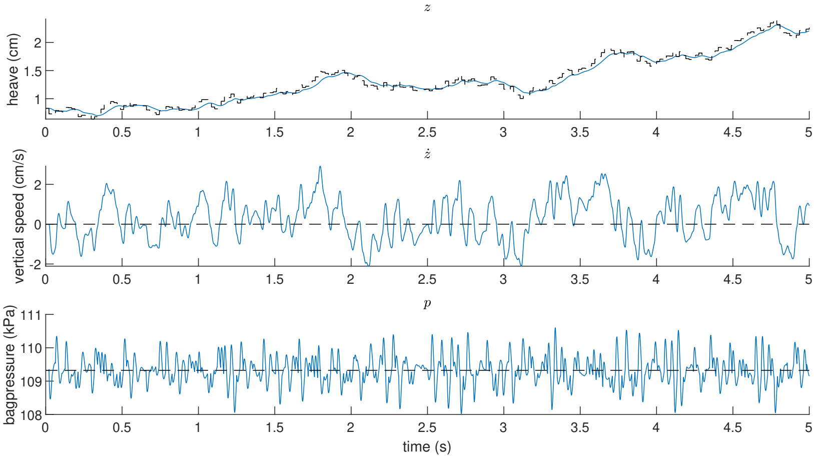
$$A_e = L(z + w_{height}) + W w_{gap} \tag{5d}$$

$$\frac{\partial P}{\partial z} = \frac{p - p_{atmo}}{\ell} \tag{5e}$$

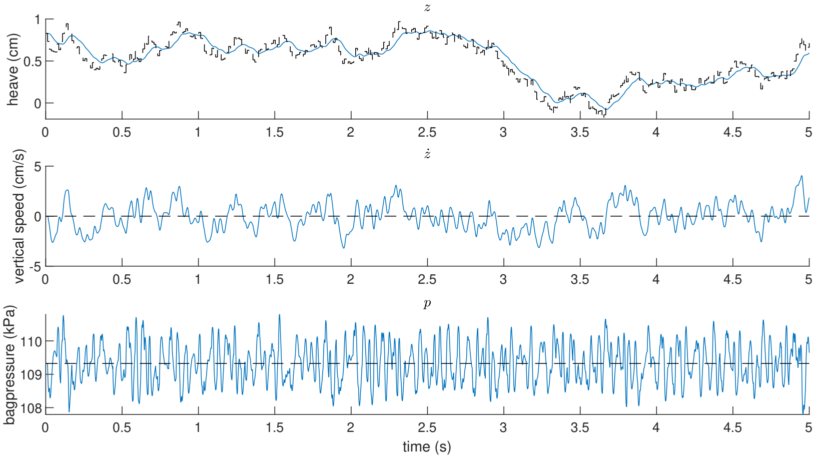
60 mph simulation



400 mph simulation



700 mph simulation



Issues with Model

Breaks down in low pressure

Full model requires 4 skates, include pitch and roll

ODE approximation to Navier-Stokes

Neglects spatial pressure dependence

Ignores bag tearing effects

Hybrid contact dynamics

Couette/Poiseuille flow lowers outside pressure

Conclusion

Simple model of Air Cushion Vehicle

Stable in atmospheric pressure

Tendency towards instability at low pressure

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